Undergraduate Colloquium in Mathematics

Illinois State University

2:00–2:50 PM, Thursday, October 13th, 2022

Speaker: Jonathan Merzel, Soka University

Ducci Cycles

Abstract:

Let $n > 1$ be an integer and let $T : \mathbb{R}^n \to \mathbb{R}^n$ be given by

$$T(a_1, \ldots, a_n) = (|a_1 - a_2|, \ldots, |a_{n-1} - a_n|, |a_n - a_1|).$$

We are interested in the dynamical phenomena when this map is iterated; for $\mathbf{v} \in \mathbb{R}^n$ the sequence $\{T^k(\mathbf{v}) | k \in \mathbb{N}\}$ is called a Ducci sequence or Ducci n-game. It turns out that to understand periodic Ducci sequences, it is sufficient to work mod 2. So we define a reduced Ducci n-game to consist of the iterates of a vector $\mathbf{v} \in \mathbb{Z}_2^n$ under the linear operator $T : \mathbb{Z}_2^n \to \mathbb{Z}_2^n$ given by

$$T(a_1, \ldots, a_n) = (a_1 + a_2, \ldots, a_{n-1} + a_n, a_n + a_1).$$

A cycle is a set of vectors $\{\mathbf{v}, T(\mathbf{v}), \ldots, T^{k-1}(\mathbf{v})\}$ with $T^k(\mathbf{v}) = \mathbf{v}$, with the smallest such k (i.e., the cardinality of the set) called the period of the cycle, or of $\mathbf{v}$. We will show how to construct a generating polynomial encoding the numbers of cycles of each different period.

Biography: Jon Merzel did his Ph.D. at University of California, Berkeley under the direction of T-Y Lam; his dissertation concerned ordered fields, quadratic forms and valuation theory. Areas of current interest include valuation theory, recreational mathematics, and (as a latecomer) ring theory and some algebraic questions connected to networks and graphs. His previous work on Ducci sequences consisted of several papers coauthored with Ron Brown, now professor emeritus, University of Hawaii.

This presentation will be on zoom and all are invited.

ZOOM link:
http://https://illinoisstate.zoom.us/j/94858996931?pwd=KzlWZEV6d1d1c3Mvd2JqNEJqOWhtdz09
Meeting ID: 948 5899 6931
Passcode: ISU