

# Discrete Mathematics Seminar

Illinois State University

2:00–2:50 pm, April 14

Speaker: Amin Bahmanian, Illinois State University

## Extending Colorings of Triples Using Fewest Number of Colors

Let  $G$  be a multi-forest. Marcotte and Seymour established necessary and sufficient conditions under which a coloring of  $F \subseteq G$  can be extended to a coloring of  $G$ . Now, let  $X, Y$  be finite sets with  $X \subseteq Y$  and  $|Y| \equiv 0 \pmod{3}$ . When can we extend a coloring of  $\binom{X}{3}$  (all triples on a ground set  $X$ ) to a coloring of  $\binom{Y}{3}$  using fewest possible number of colors? It is necessary that the number of triples of each color in  $\binom{X}{3}$  is at least  $|X| - |Y| + |Y|/3$ . We establish a necessary and sufficient condition in terms of list coloring complete multigraphs. Using Häggkvist-Janssen's bound, we show that the number of triples of each color being at least  $|X|/2 - |Y|/6$  is sufficient. Finally we show that if  $|Y| \geq 3|X|$ , then any  $q$ -coloring of any subset of  $\binom{X}{3}$  can be extended to a  $\binom{|Y|-1}{2}$ -coloring of  $\binom{Y}{3}$  as long as  $q \leq \binom{|Y|-1}{2} - \binom{|X|-1}{2}$ .

