Discrete Mathematics Seminar

Illinois State University

2:00–2:50 pm, February 17

Speaker: Murong Xu, The University of Scranton

An extremal problem of arc-strong connectivity of digraphs

A digraph D is strong if for any pair of vertices $u, v \in V(D)$, D always contains a (u, v)-dipath. The arc-strong connectivity of a digraph D, denoted by $\lambda(D)$, is the minimum number of arcs whose removal results in a non-strong digraph. If we just count the number of arcs in a digraph, can we predict that D contains a subdigraph with high arc-strong connectivity? We define $\overline{\lambda}(D) = \max\{\lambda(H) : H \subseteq D\}$. Given an integer $k > 0$, a strict digraph D is k-maximal if $\overline{\lambda}(D) \leq k$ but adding any arc which is not in D will surely create a subdigraph with arc-strong connectivity at least $k + 1$. Mader [Math. Ann. 1971] and Lai [JGT 1990] studied the extremal size of undirected k-maximal graphs. We determine that if D is a k-maximal digraph on $n > k$ vertices, then

$$\binom{n}{2} + (n-1)k + \lfloor \frac{n}{k+2} \rfloor \left(1 + 2k - \binom{k+2}{2}\right) \leq |A(D)| \leq k(2n-k-1) + \binom{n-k}{2}.$$

Consequently, if $A(D)| > k(2n-k-1) + \binom{n-k}{2}$, then D must have a nontrivial subdigraph H such that the arc-strong connectivity of H is at least $k + 1$. In this talk, I will introduce this extremal problem and the constructive characterization of a family of digraphs with the extremal size. Furthermore, the current progress on this project will be discussed.