

## ISU Algebra Seminar

**Date:** Thursday, September 19, 2019

**Time:** 1-1:50 pm

**Location:** STV 347B

**Speaker:** Fusun Akman

**Talk title:** Group Theory in Symmetries of Gene Regulatory Networks

**Abstract:** (Joint work with Devin Akman) *Gene Regulatory Networks (GRN)* form a flexible modeling scheme for synthesizing and analyzing the interactions of up to thousands of factors such as functional genes, enzymes, and other proteins in a cell (loosely called *genes*) that affect one another's functions in a particular unit of biological process. One example is the successfully deciphered lac operon of *E. coli* (see below).



The overall states (0 or 1; off or on; below or above a threshold) of  $n$  genes at any moment can be any one of  $N = 2^n$  binary sequences. A full description of the GRN is given by what the next state will be, given the current state. Therefore, we need to produce an *update function*  $F = (f_1, \dots, f_n)$  from the set of  $N$  states into itself, where all  $f_i$  are ordinary Boolean functions with  $n$  inputs and one output from the set  $\{0, 1\}$ . A *functional digraph* (directed graph where every vertex has exactly one outgoing edge) on  $N$  vertices is then sufficient to describe any GRN. Almost no theoretical work exists on the symmetries (automorphism groups) of these *state ("phase") spaces*. We will describe the group theoretical methods that we have used to determine the largest possible automorphism groups of phase spaces (this involves describing all subgroups of direct products of groups), and to find out what the largest such group can be when all  $n$  Boolean functions that make up  $F$  are chosen from the special class of *nested canalizing functions*. (Spoiler alert: the answers are  $S_N$  and  $S_{N-2}$  respectively, with only  $S_{N-1}$  and  $C_2 \times S_{N-2}$  in between.) Come and find out, as a bonus, why symmetries are so important!

