ISU Algebra Seminar

Title: NUMB3RS
Speaker: Fusun Akman
Location: STV 310
Time: Noon- 12:50 pm on Thursday (03/05/20)
Abstract:

What is a number system? The short answer is anything we can count, order, or measure with, preferably some extension of the natural numbers with order and operations. For the unadventurous, the standard sequence of N, Z, Q, R, C, quaternions, octonions,… does the job. Building the natural numbers according to Peano axioms is the first stage. At every subsequent stage, we use only the number set that has just been constructed (and equivalence relations when necessary) to build the next number set, all the way to the only complete ordered field R, the unordered but algebraically closed field C, then going downhill, losing first commutativity and then associativity of multiplication. Thanks to Cantor, we know that more can be done. Natural numbers can be extended to ordinal numbers to describe the order types of all well-ordered sets by adding transfinite numbers (ordinals include cardinals). So, can we fill in between ordinals to get the most perfect number system? That's how Conway created the surreal numbers, which is the ultimate ordered FIELD (caps due to the underlying class as opposed to set): surreals subsume ordinals and cardinals, as well as every ordered field, including reals and hyperreals (with the infinite and infinitesimal elements that we love in Calculus). What is more, the construction of surreals does not depend on a previously constructed set but starts from zero and creates all "numbers" by adding stuff in between. Since NUMB3RS is a topic that needs at least a semester to cover (which is what I happen to be doing in MAT 338), I will select a few of the constructions (natural numbers, ordinals, hyperreals, and surreals) and point out the common goals and techniques with a little bit of history. This talk is a survey geared towards undergraduates and number lovers of all stripes. (Background? Sets, equivalence relations, and equivalence classes.)