Eulerian hypergraphs

Let $k \geq 3$ and $H$ be a $k$-uniform hypergraph. An Euler tour in $H$ is an alternating sequence $v_0, e_1, v_1, e_2, v_2, \ldots, v_{m-1}, e_m, v_m = v_0$ of vertices and edges in $H$ such that each edge of $H$ appears in this sequence exactly once and $v_{i-1}, v_i \in e_i, v_{i-1} \neq v_i$, for every $i = 1, 2, ..., m$. Lonc and Naroski showed that for $k \geq 3$, the problem of determining if a given $k$-uniform hypergraph has an Euler tour is NP-complete. For $1 \leq \ell \leq k$, the minimum $\ell$-degree of $H = (V, E)$ is $\delta_{\ell}(H) = \min_{S \subseteq V, |S| = \ell} |\{e \mid S \subseteq e, e \in E\}|$. Šajna and Wagner showed that every 3-uniform hypergraph $H = (V, E)$ with $\delta_2(H) \geq 2$ admits an Euler tour. As a consequence, every $k$-uniform hypergraph $H = (V, E)$ with $\delta_{k-1}(H) \geq 2$ has an Euler tour. We investigate the existence of Euler tour in $k$-uniform hypergraphs for $k \geq 4$ under $\ell$-degree conditions with $1 \leq \ell \leq k-2$.

In particular, for $k \geq 4$, we show that every $k$-uniform hypergraph $H = (V, E)$ with $\delta_2(H) \geq k$ or $\delta_{k-2}(H) \geq 4$ and with $|V| \geq \frac{k^2}{2} + \frac{k}{2}$ admits an Euler tour.

This is Joint work with Amin Bahmanian.