Discrete Mathematics Seminar

Illinois State University

2:00–2:50 pm, August 22@ STV 120

Speaker: Songling Shan, Illinois State University

Eulerian hypergraphs

Let $k \geq 3$ and H be a k-uniform hypergraph. An Euler tour in H is an alternating sequence $v_0, e_1, v_1, e_2, v_2, \cdots, v_{m-1}, e_m, v_m = v_0$ of vertices and edges in H such that each edge of H appears in this sequence exactly once and $v_{i-1}, v_i \in e_i, v_{i-1} \neq v_i$, for every $i = 1, 2, ..., m$. Lonc and Naroski showed that for $k \geq 3$, the problem of determining if a given k-uniform hypergraph has an Euler tour is NP-complete. For $1 \leq \ell \leq k$, the minimum ℓ-degree of $H = (V, E)$ is $\delta_\ell(H) = \min_{S \subseteq V, |S| = \ell} |\{e \,|\, S \subseteq e, e \in E\}|$. Šajna and Wagner showed that every 3-uniform hypergraph $H = (V, E)$ with $\delta_2(H) \geq 2$ admits an Euler tour. As a consequence, every k-uniform hypergraph $H = (V, E)$ with $\delta_{k-1}(H) \geq 2$ has an Euler tour. We investigate the existence of Euler tour in k-uniform hypergraphs for $k \geq 4$ under ℓ-degree conditions with $1 \leq \ell \leq k-2$. In particular, for $k \geq 4$, we show that every k-uniform hypergraph $H = (V, E)$ with $\delta_2(H) \geq k$ or $\delta_{k-2}(H) \geq 4$ and with $|V| \geq \frac{k^2}{2} + \frac{k}{2}$ admits an Euler tour.

This is Joint work with Amin Bahmanian.