Discrete Mathematics Seminar

Illinois State University

2:00–2:50 pm, August 29@ STV 121

Speaker: Papa A. Sissokho, Illinois State University

Geometric Characterization of the minimal nonnegative solutions of a linear Diophantine Equation

Let $\mathbf{a} = (a_1, \ldots, a_n)$ and $\mathbf{b} = (b_1, \ldots, b_m)$ be vectors whose entries are distinct positive integers. Let $\mathcal{S}(\mathbf{a}, \mathbf{b})$ denote the set of all nonnegative solutions $(\mathbf{x}, \mathbf{y})$, where $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_m)$, of the linear Diophantine equation $x_1a_1 + \ldots + x_na_n = y_1b_1 + \ldots + y_mb_m$. A solution is called *minimal* if it cannot be written as the sum of two nonzero solutions in $\mathcal{S}(\mathbf{a}, \mathbf{b})$. The set of all minimal solutions, denoted by $\mathcal{H}(\mathbf{a}, \mathbf{b})$, is called the *Hilbert basis* of the set of all solutions $\mathcal{S}(\mathbf{a}, \mathbf{b})$. For $1 \leq i \leq n$ and $1 \leq j \leq m$, the solution $\mathbf{g}_{i,j} = (b_j\mathbf{e}_i, a_i\mathbf{e}_{n+j})$ of the above Diophantine equation, where $\mathbf{e}_k$ is the kth standard unit vector of $\mathbb{R}^{n+m}$, is called a *generator*. In this talk, we discuss general notions related to Hilbert bases and their applications. In particular, we will discuss a recent result which shows that every minimal solution in $\mathcal{H}(\mathbf{a}, \mathbf{b})$ is a *convex combination* of the generators and the zero-solution. This confirms a conjecture of Henk–Weismantel and, independently, Hosten–Sturmfels.