

Topic: Functional notation

Background: Given functions $f(x)$ and $g(x)$, an expression for the function $f(g(x))$ is obtained by substituting $g(x)$ for each occurrence of x in the expression for $f(x)$.

Illustrative Examples:

- (1) Let $f(x) = 3x^2 - 7x + 2$. Find an expression for $f(2x - 1)$ in simplified form.

Solution:

$$f(2x - 1) = 3(2x - 1)^2 - 7(2x - 1) + 2 = 3(4x^2 - 4x + 1) - 14x + 7 + 2 = 12x^2 - 26x + 12.$$

- (2) Let $f(x) = 3x^2 + 2x + 7$. For $h \neq 0$ find an expression for $\frac{f(x+h)-f(x)}{h}$ in simplified form.

Solution:

$$\begin{aligned} f(x+h) &= 3(x+h)^2 + 2(x+h) + 7 \\ &= 3(x^2 + 2xh + h^2) + 2x + 2h + 7 \\ &= 3x^2 + 6xh + 3h^2 + 2x + 2h + 7 \end{aligned}$$

Hence,

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(3x^2 + 6xh + 3h^2 + 2x + 2h + 7) - (3x^2 + 2x + 7)}{h} \\ &= \frac{6xh + 3h^2 + 2h}{h} \\ &= 6x + 3h + 2 \end{aligned}$$

- (3) Let $f(x) = 7x^3 + 9$ and let $g(y) = y^2$. Find an expression for $f(g(z))$.

Solution:

$$f(g(z)) = f(z^2) = 7(z^2)^3 + 9 = 7z^6 + 9.$$

- (4) Let $f(x) = 4 \ln(x) + x^2 + 7$. Find an expression for $f(e^x)$ in simplified form.

Solution:

$$f(e^x) = 4 \ln(e^x) + (e^x)^2 + 7 = 4x + e^{2x} + 7.$$

- (5) Let $f(x) = 7x - 30$. Find an expression for $f\left(\frac{x+30}{7}\right)$.

Solution:

$$f\left(\frac{x+30}{7}\right) = 7\left(\frac{x+30}{7}\right) - 30 = x.$$