

Topic: Lines

Background:

Slope m of line through points $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of line through point $P(x_1, y_1)$ and slope m :

$$y - y_1 = m(x - x_1)$$

Equation of line through points $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

Equation of line with slope m and y -intercept c :

$$y = mx + c$$

Distance d between points $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Illustrative Examples:

- (1) Find the equation of the line through the point $(3,7)$ and with slope $-\frac{3}{4}$.

Solution:

$$y - 7 = \left(-\frac{3}{4}\right)(x - 7).$$

- (2) Find the y -intercept of the line through the point $(3,7)$ and with slope $-\frac{3}{4}$.

Solution:

From problem (1) above, the equation of the line is

$$y - 7 = \left(-\frac{3}{4}\right)(x - 7).$$

The y -intercept of the line is the y -coordinate of the point where the line intersects the y -axis and hence it is the value of y in the equation of the line when $x = 0$. Setting $x = 0$ in the equation of the line and solving for y gives

$$y = \left(-\frac{3}{4}\right)(0 - 7) + 7 = \frac{49}{4}.$$

- (3) Find the equation of the line through the points (8,3) and (6, -9).

Solution:

$$y - 3 = \left(\frac{(-9)-3}{6-8}\right)(x - 8) = 6(x - 8) \text{ or } y = 6x - 45.$$

- (4) Let L be the line whose equation is $y = 7x + 30$. Find the equation of a line which is parallel to L and that passes through the point (5,8).

Solution:

Slope of required line = slope of L = 7 and, the required line passes through (5,8).

Hence the equation of the required line is

$$y - 8 = 7(x - 5) \text{ or } y = 7x - 27.$$

- (5) Find the perimeter of the triangle with vertices A(3,5), B(4,7), and C(12,3).

Solution:

Perimeter of the triangle = distance between A and B + distance between B and C + distance between A and C.

Hence,

$$\begin{aligned} \text{Perimeter} &= \sqrt{(4-3)^2 + (7-5)^2} + \sqrt{(12-4)^2 + (3-7)^2} + \sqrt{(12-3)^2 + (3-5)^2} \\ &= \sqrt{5} + \sqrt{80} + \sqrt{85} \end{aligned}$$