Solutions

Integration by Parts

Examples

$\int x^2 e^x dx$ Let $u = x^2$ $dv = e^x dx$

then $du = 2x\,dx$, $v = e^x$

So

$$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx$$

I.B.P. again:

Let $u = 2x$ $dv = e^x dx$

then $du = 2\,dx$ $v = e^x$

So

$$\int x^2 e^x dx = x^2 e^x - \left[2x \cdot e^x - \int 2 e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2\int e^x dx$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$\int_0^1 3x^4 e^{5x}\,dx$$

Use TABULAR I.B.P.

derive to 0

u		dv
$3x^4$	$+$	$e^{5x}dx$
$12x^3$	$-$	$\frac{1}{5}e^{5x}$
$36x^2$	$+$	$\frac{1}{25}e^{5x}$
$72x$	$-$	$\frac{1}{125}e^{5x}$
72	$+$	$\frac{1}{625}e^{5x}$
0	$-$	$\frac{1}{3125}e^{5x}$

keep integrating

So...

$$\int_0^1 3x^4 e^{5x}\,dx = \Big[\left(3x^4\left(\tfrac{1}{5}e^{5x}\right)\right) - \left(12x^3\left(\tfrac{1}{25}e^{5x}\right)\right) + \left(36x^2\left(\tfrac{1}{125}e^{5x}\right)\right) - \left(72x\left(\tfrac{1}{625}e^{5x}\right)\right) + \left(72\left(\tfrac{1}{3125}e^{5x}\right)\right)\Big]_0^1$$

$$= \frac{3}{3125}e^{5x}\Big[625x^4 - (4x^3)(125) + (12x^2)(25) - (24x)(5) + (24)\Big]_0^1$$

$$= \frac{3}{3125}e^5\left(625 - 500 + 300 - 120 + 24\right) - \left(\frac{3}{3125}(24)\right) = \frac{3}{3125}\left(329e^5 - 24\right)$$

$$\int_1^{e^2} x \ln x \, dx \qquad u = \ln x \quad dv = x\,dx$$

$$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$$

so $$\int_1^{e^2} x \ln x \, dx = \frac{1}{2}\ln x \cdot x^2 - \frac{1}{2}\int_1^{e^2} x \, dx$$

$$= \frac{(\ln x)x^2}{2} - \left(\frac{1}{2}\right)\left(\frac{x^2}{2}\right)\Big]_1^{e^2}$$

$$= \left(\frac{e^4}{2}\ln(e^2) - \frac{1}{4}(e^2)^2\right) - \left(\frac{1}{2}\ln(1) - \frac{1}{4}(1)^2\right)$$

$$= e^4 - \frac{e^4}{4} - 0 + \frac{1}{4}$$

$$= \frac{1}{4} + \frac{3e^4}{4} = \frac{3}{4}e^4 + \frac{1}{4}$$

$$\int \sin(2x)e^x\,dx$$

Let $u = \sin(2x)$, $dv = e^x dx$

then $du = \cos(2x)\cdot 2$, $v = e^x$

$$= e^x \sin(2x) - 2\int \cos(2x)e^x\,dx$$

I.B.P. again...

$u = \cos(2x)$ $dv = e^x dx$

so $du = (-\sin 2x)(2)\,dx$ $v = e^x$

$= -2\sin(2x)\,dx$

$$\int \sin(2x)e^x\,dx = e^x(\sin(2x)) - 2\left[e^x\cos(2x) + 2\int \sin(2x)e^x\,dx\right]$$

$$= e^x\sin(2x) - 2e^x\cos(2x) - 4\int \sin(2x)e^x\,dx$$

<u>Note</u>: Let $A = \int \sin(2x)e^x\,dx$; we have

$$A = e^x\sin(2x) - 2e^x\cos(2x) - 4A$$

so $5A = e^x\sin(2x) - 2e^x\cos(2x)$

$\Rightarrow A = \int \sin(2x)e^x\,dx = \frac{1}{5}(e^x)\left[\sin(2x) - 2\cos(2x)\right] + C$

$$\int 3x \sin(2x)\,dx$$

Let

$u = 3x, \quad dv = \sin(2x)\,dx$

$du = 3\,dx, \quad v = -\frac{1}{2}\cos(2x)$

so

$$\int 3x \sin(2x)\,dx = -\frac{3x}{2}\cos(2x) + \frac{3}{2}\int \cos(2x)\,dx$$

$$= -\frac{3}{2}x\cos(2x) + \left(\frac{3}{2}\right)\left(\frac{1}{2}\sin(2x)\right) + C$$

$$= -\frac{3}{4}\left(2x\cos(2x) - \sin(2x)\right) + C$$

$$\int \frac{\ln x}{x^5}\,dx \qquad \text{Let} \quad u = \ln x \qquad dv = \frac{1}{x^5}\,dx$$

$$du = \frac{1}{x}\,dx \qquad v = -\frac{1}{4x^4}\,dx$$

so

$$\int \frac{\ln x}{x^5}\,dx = -\frac{1}{4}\cdot\frac{\ln|x|}{x^4} + \frac{1}{4}\int \frac{1}{x^5}\,dx$$

$$= \frac{\ln|x|}{4x^4} + \left(\frac{1}{4}\right)\left(-\frac{1}{4x^4}\right) + C$$

$$= \frac{\ln|x|}{4x^4} - \frac{1}{16x^4} + C$$

$$= \frac{1}{16x^4}\left(4\ln|x| - 1\right) + C$$