Greetings!

I know you remember that when we began this chapter, the first task we explored was this:

Carry out the division:

$$\frac{1}{1-x} = 1 \div (1-x)$$

and we generated this strange expression:

$$1 + x + x^2 + x^3 + \cdots$$

We called the last expression a ***POWER SERIES***. It's an ***infinite polynomial***.

We then connected those two expressions, graphing both, to show that the infinite polynomial could be used to stand in for $\frac{1}{1-x}$.

Finally, we showed that this stand-in role could be used ONLY when $-1 < x < 1$. We eventually called this last bit of info the ***INTERVAL of CONVERGENCE***.

Of course, you now know that

$$1 + x + x^2 + x^3 + \cdots$$

is not only a Power Series, it is a ***GEOMETRIC*** power series with the common ratio $r = x$. Knowing the common ratio provides another way to determine the interval of convergence for the infinite series:

For any geometric series we need

$$|r| < 1$$

So, here, for $1 + x + x^2 + x^3 + \cdots$, we need

$$|x| < 1$$

and that leads to

$$-1 < x < 1$$

Which is just what we had determined at the beginning of the chapter!

We now return to this geometric power series stand-in function,

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \text{ for } -1 < x < 1$$

We use that to create many new stand-in functions.

For instance, knowing that

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \text{ for } -1 < x < 1$$

What power series, do you suppose, represents

$$g(x) = \frac{2}{1-x}?$$

Make a connection between $g(x)$ and $f(x)$:

$$g(x) = \frac{2}{1-x} = 2 \cdot \frac{1}{1-x} = 2(1 + x + x^2 + x^3 + \cdots) = 2 + 2x + 2x^2 + 2x^3 + \cdots$$

That is,

$$g(x) = \frac{2}{1-x} = 2 + 2x + 2x^2 + 2x^3 + \cdots = \sum_{k=0}^{\infty} 2x^k$$

Before reading further, try to connect

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

to

$$h(x) = \frac{5}{3-x}$$

We have

$$h(x) = \frac{5}{3-x} = \frac{5}{3} \cdot \frac{1}{1 - \frac{1}{3}x}$$

Here, $\frac{1}{1-\frac{1}{3}x}$ looks a lot like $\frac{1}{1-x}$. It must be a geometric series, with first term $a_1 = 1$ and common ratio $r = \frac{1}{3}x$.

Note that with

$$r = \frac{1}{3}x$$

we must have

$$\left|\frac{1}{3}x\right| < 1$$

and that leads to the interval of convergence for this power series to be

$$-3 < x < 3$$

So we have a geometric power series stand in for $1\frac{1}{1-\frac{1}{3}x}$:

$$\frac{1}{1-\frac{1}{3}x} = 1 + \frac{1}{3}x + \left(\frac{1}{3}x\right)^2 + \left(\frac{1}{3}x\right)^3 + \cdots \text{ for } -3 < x < 3$$

And with that we can finish writing a stand-in for $h(x)$:

$$h(x) = \frac{5}{3-x} = \frac{5}{3}\cdot\frac{1}{1-\frac{1}{3}x} = \frac{5}{3}\cdot\left[1 + \frac{1}{3}x + \left(\frac{1}{3}x\right)^2 + \left(\frac{1}{3}x\right)^3 + \cdots\right] \text{ for } -3 < x < 3$$

$$= \frac{5}{3} + \frac{5}{3}\cdot\frac{1}{3}x + \frac{5}{3}\cdot\left(\frac{1}{3}x\right)^2 + \frac{5}{3}\cdot\left(\frac{1}{3}x\right)^3 + \cdots$$

$$= \frac{5}{3} + \frac{5}{9}x + \frac{5}{27}x^2 + \frac{5}{81}x^3 + \cdots$$

And, finally,

$$h(x) = \frac{5}{3} + \frac{5}{9}x + \frac{5}{27}x^2 + \frac{5}{81}x^3 + \cdots = \sum_{k=0}^{\infty}\frac{5x^k}{3^{k+1}} \text{ for } -3 < x < 3$$

You are now ready to tackle the next WebAssign tasks! These are due in a couple days and include some extra-credit tasks.

If you need more examples and explanations, go to:
http://tutorial.math.lamar.edu/Classes/CalcII/PowerSeriesandFunctions.aspx