MAT 145: Quiz #10 (Part 1: 26 points)

Name ____________________ **Section** ______________ **Score** ______________

Part I: Do Not Use Any Calculator or Computer Tools!

State the derivative of each function. Each derivative should start with $y' =$ or $\frac{dy}{dx} =$. ***Do not simplify.*** (1 pt each)

1. $y = 10x^4$ $\qquad y' = 40x^3$

2. $y = e^{3x}$ $\qquad dy/dx = 3e^{3x}$

3. $y = 8x^9 - 5x^8 + 4e^7$ $\qquad dy/dx = 72x^8 - 40x^7$

4. $y = 11^x$ $\qquad y' = (\ln 11) \cdot 11^x$

5. $y = \tan(x)$ $\qquad y' = \sec^2(x)$

6. $y = \ln(x)$ $\qquad y' = \frac{1}{x}$

7. $y = \sin\left(\frac{1}{2}x\right)$ $\qquad y' = \frac{1}{2}\cos\left(\frac{1}{2}x\right)$

8. $y = x^2\cos(x)$ Product:

   $dy/dx = (2x)\cos(x) + x^2(-\sin x)$

   $= 2x\cos x - x^2 \sin x$

   $dy/dx = 2x\cos x - x^2 \sin x$

9. $y = \frac{1}{x} = x^{-1}$; so... $\frac{dy}{dx} = (-1)x^{-2} = \frac{-1}{x^2}$

   $dy/dx = -\frac{1}{x^2}$

10. $y = \frac{2 - x^3}{x^2 + 1}$ Quotient:

    $y' =$

    $\frac{(-3x^2)(x^2+1) - (2-x^3)(2x)}{(x^2+1)^2}$

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Part I (continued): Do Not Use Any Calculator or Computer Tools!

For #11 and #12, determine the derivative of y with respect to x. Use **appropriate notation** for each derivative, **show all required steps**, and **express the derivative explicitly as a function of** x**, if possible**.

11. (3 pts) $3y^2 - 2xy = 5$

implicit dif: $6y \cdot \frac{dy}{dx} - \left[(2)\cdot y + (2x)\cdot \frac{dy}{dx}\right] = 0$

$\frac{dy}{dx}(6y - 2x) = 2y$

$\Rightarrow dy/dx = \frac{2y}{(6y-2x)}$

$\frac{dy}{dx} = \frac{2y}{6y-2x} = \frac{y}{3y-x}$

12. (3 pts) $y = (x)^{\tan x}$

$\frac{dy}{dx} = \left(x^{\tan x}\right)\left(\sec^2 x \cdot \ln x + \frac{\tan x}{x}\right)$

use log. dif.: $\ln y = \ln\left(x^{\tan x}\right)$

$\Rightarrow \ln y = (\tan x)(\ln x)$

imp dif: $\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = (\sec^2 x)\ln x + \tan x \cdot \frac{1}{x}$

$\Rightarrow dy/dx = (y)\left[\sec^2 x \cdot \ln x + \frac{\tan x}{x}\right]$

13-22 (1 pt each)

13) $\lim_{x\to -4} f(x)$ $\underline{\ 0\ }$

14) $\lim_{x\to 2^+} f(x)$ $\underline{\ -\infty\ }$

15) $\lim_{x\to 4} f(x)$ $\underline{\ 5/2\ }$

16) $\lim_{x\to 0^-} f(x)$ $\underline{\ 1\ }$

17) $f(2)$ $\underline{\ 4\ }$

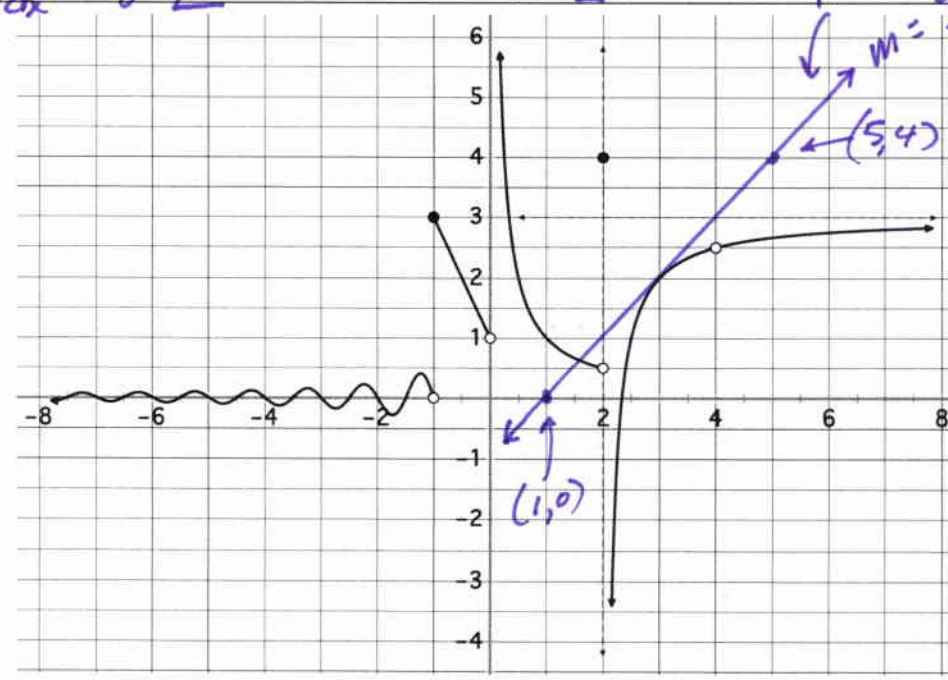
18) $f(4)$ $\underline{\ \text{DNE}\ }$

19) $\lim_{x\to -\infty} f(x)$ $\underline{\ 0\ }$

20) $\lim_{x\to -1^+} f(x)$ $\underline{\ 3\ }$

21) $f'(-0.5)$ $\underline{\ -2\ }$

22) $f'(3)$ $\underline{\ 1\ }$



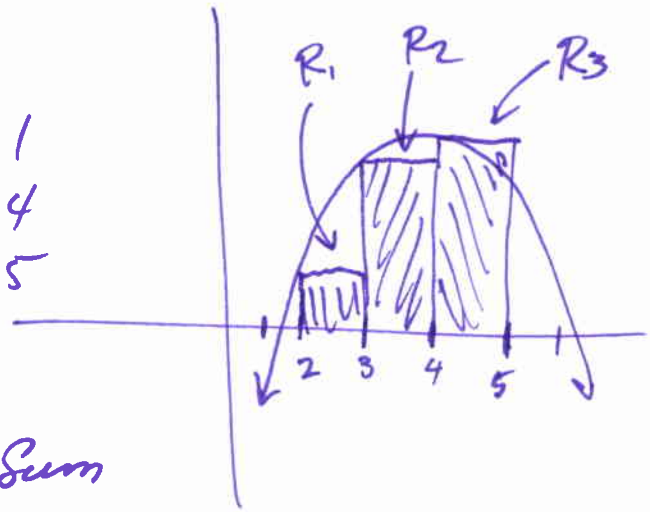
SOLUTION GUIDE

MAT 145: Quiz #10 (Part 2: 9 points)

Name ____________________ **Section** ________________ **Score** ________________

23. Calculate the exact value of L_3, a left-endpoint Riemann Sum approximation for $\int_2^5 5-(x-4)^2$, the area trapped between the curve $y=5-(x-4)^2$ and the x-axis, using three subdivisions on the interval $2 \le x \le 5$. (9 pts total, including sketch of function with accurately sketch rectangles (4 pts), correct determination of Δx (1 pt), exact-area calculations of approximating rectangles (3 pts), and correct exact value for L_3 (1 pt))

$\Delta x = \frac{5-2}{3} = \frac{3}{3} = 1$



Rec #1 – AREA: $\Delta x \cdot f(2) = (1)(1) = 1$

#2 – : $\Delta x \cdot f(3) = (1)\cdot(4) = 4$

#3 – : $\Delta x \cdot f(4) = (1)(5) = 5$

TOTAL AREA: $1+4+5 = 10$

$L_3 = 10$, the Riemann Sum for $y = 5-(x-4)^2$ on $2 \le x \le 5$, $n=3$ subdivisions, left endpts

____ **4 pts**: sketch of function with accurately sketch rectangles;

____ **1 pt**: correct determination of Δx;

____ **3 pts**: exact-area calculations of approximating rectangles;

____ **1 pt**: correct exact value for L_3.

Name SOLUTION GUIDE

BONUS: 4 points!

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The velocity of an object moving along a line is given by $v(t) = 6t - 18\ ^{m}/_{s}$. Calculate the ***total distance traveled*** by the object on the interval $0 \le t \le 5$ seconds. Include appropriate calculus-based evidence. Check the units of measure you use in your response.

The complication here is that the object may change direction on $0 \le t \le 5$. Solve $v(t)=0$ to check that: $v(t)=0 \Rightarrow 6t-18=0 \Rightarrow t=3$.

Thus, we need distance traveled on $0 \le t \le 3$ and on $3 \le t \le 5$, use absolute value of those measures and add

$$\text{TOTAL DISTANCE TRAVELED} = \left| \int_0^3 v(t)\,dt \right| + \left| \int_3^5 v(t)\,dt \right|$$

$$= \left| \int_0^3 6t-18\,dt \right| + \left| \int_3^5 6t-18\,dt \right|$$

$$= \left| 3t^2-18t \Big]_0^3 \right| + \left| 3t^2-18t \Big]_3^5 \right|$$

$$= \left| -27 \right| + \left| 12 \right| = 27+12 = 39$$

meters