MAT 145: Test #4 – Part II (30 points)

Part 2: Calculator OK!

Name ________________________ **Calculator Used** ____________ **Score** ____________________

19. Lauren calculated the exact value of $\int_1^2 \frac{1}{3}x^3\,dx$ using the **Fundamental Theorem of Calculus**. She also calculated a **Riemann Sum approximation** of $\int_1^2 \frac{1}{3}x^3\,dx$ using $\boldsymbol{L_3}$ (left-hand rule with 3 rectangles used to estimate the area between the curve and the x-axis, $1 \le x \le 2$). She correctly calculated $\int_1^2 \frac{1}{3}x^3\,dx = \frac{5}{4}$ and $L_3 = \frac{8}{9}$. (10 pts)

(a) Sketch an accurate drawing to represent Lauren's $\boldsymbol{L_3}$ approximating rectangles. Include and label the axes and the function. Indicate the exact value of Δx. No other calculations are required. (6 pts)

(b) Write a brief paragraph to clearly and concisely explain how your diagram in (a) supports Lauren's numerical calculation that $L_3 < \int_1^2 \frac{1}{3}x^3\,dx$. (4 pts)

20. A rancher has 600 meters of fencing and wants to enclose a rectangular region that borders a utility shed. ***She needs no fence along the utility shed***. Determine the dimensions of the region so that the maximum possible area is enclosed.

In your response, ***identify all variables*** and clearly indicate what each variable represents. ***Include a sketch***, if appropriate, labeled with variables or algebraic expressions that include variables. Clearly ***state the domain of the independent variable***. ***Show all calculations*** or calculator inputs and outputs, and ***justify*** that the dimensions you found do, indeed, lead to a global maximum. Show clear and complete ***evidence that you used calculus*** to solve this problem and that you considered all situations. ***Clearly state your response to the question, using appropriate units***. (10 pts)

____ **1 pt**: variables identified/described;	
____ **2 pts**: labeled sketch;	
____ **1 pt**: domain of independent variable;	
____ **2 pts**: calculations shown;	TOTAL
____ **1 pt**: justify maximum;	
____ **2 pts**: calculus evidence;	
____ **1 pt**: correct solution with units	/10

21. An object is dropped from the Observation Deck on the 86^{th} floor of the Empire State Building in Manhattan (NYC). This observation deck is 320 meters above the ground. Assume that acceleration due to gravity is $-9.8\ m/s^2$ and that the initial velocity of the object is $0\ m/s$.

a) Determine a position function, $s(t)$, to represent the position of the object above the ground, at time t, t in seconds. Show evidence to justify your response. ***Include appropriate units*** (5 pts)

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b) Calculate the length of time required for the object to reach the ground. Round to the nearest ***hundredth of a unit (two decimals).*** Show evidence to justify your response. ***Include appropriate units***. (5 pts)

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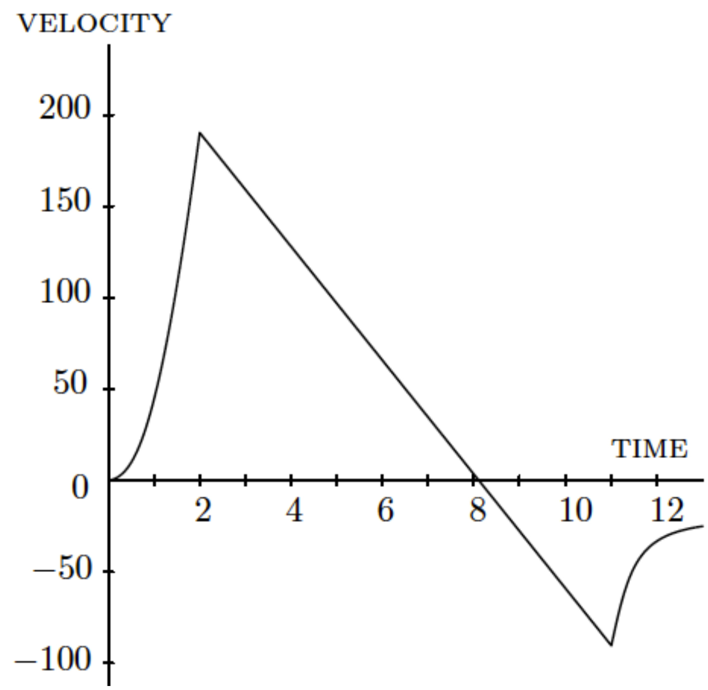
BONUS!

When a model rocket is launched, the propellant (fuel) burns for a few seconds, accelerating the rocket upward. After burnout (all fuel consumed), the rocket coasts upward for a while and then begins to fall. A small explosive charge pops out a parachute shortly after the rocket starts falling. The parachute slows the rocket to keep the rocket from breaking when it lands. The figure here shows the velocity, in feet per second, from the beginning to the end of the model rocket's flight. (2 pts each)

(A) How fast was the rocket climbing when the engine stopped? Explain.
(B) At what time did the rocket reach its highest point? What was the rocket's velocity at this time? Explain.
(C) How long did the rocket fall before the parachute opened? Explain.

Each response should be clear, concise, and accurate and include appropriate calculus justification. Feel free to mark up, label, and refer to the figure as you respond.



Calculus I
MAT 145
Test #4: 50 points

Evaluation Criteria

<u>Part I: No Calculators</u> (20 points)

1 – 7: 1 pt each

8: 3 pts

9 – 18: 1 pt each

<u>Part II: Calculators May Be Used</u> (30 points)

19: 10 pts: (a) 6 pts: Show accurate and correctly labeled sketch that includes rectangles and the function; correct determination of Δx; (b) 4 pts: Clear and concise explanation to justify $L_3 < \int_1^2 \frac{1}{3} x^3 \, dx$

20: 10 pts: Show sketch, define variables, provide domain, show calculus-based computations, justify the optimization, and provide correct solution, including units.

21: 10 pts: (a) 5 pts: Correct determination of $s(t)$ with appropriate evidence and correct units; (b) 5 pts: Correct determination of requested time, rounded as indicated; appropriate justification and use of units

<u>Bonus: Calculators May Be Used</u> (6 points)

Each question is worth 2 points. To receive any credit on any question, your numerical response must be correct and correctly labeled and include an accurate calculus-based explanation.