MAT 145 Section 2 Fall Semester 1999

Semester Exam Information

Monday 13 December 1999
Stevenson Hall 229
8:00-10:40 pm

The exam will consist of two parts.

Part I: You may not use any calculator tools.

- Carry out first step in determining derivatives of functions.
- Complete implicit differentiation and logarithmic differentiation
- Determine definite and indefinite integrals.
- Calculate limits of functions.
- Determine an equation for the tangent line to a curve.
- Solve a problem involving position, velocity, acceleration.

Part II: You may use a calculator and will find it helpful for many of the problems.

- Know and apply the limit definition of the derivative.
- Know and apply properties involving limits, continuity, differentiation, and integration
- Solve problems involving optimization, related rates, graphical characteristics of functions,area under a curve, velocity/position, rate of change/total change, and similar problems encountered during the course.

MAT 145
First-Semester Calculus
Semester Exam

Monday
13 Dec 1999
STV 229
8:00pm — 10:40 pm

Name ______________________________

Social Security Number ______________________

Part I (No Calculators)

Point Value	2	2	2	2	2	3	3	3	4	5	2	2	2	2	4
Question #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Points Earned															

Part II (Calculators Okay)

Point Value	5	2	2	2	2	3	3	3	3	3	2	2	2
Question #	16	17	18	19	20	21	22	23	24	25	26	27	28
Points Earned													

Point Value	2	3	3	3	2	3	3	2	4	3	3	3
Question #	29	30	31	32	33	34	35	36	37	38	39	40
Points Earned												

Point Value	4	4	2	2	3	3	4
Question #	41	42	43	44	45	46	47
Points Earned							

40	90
Part I	**Part II**

130
Total

Semester Percent	Semester Grade

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Part I: Calculators May Not Be Used

For each function in questions 1 through 5, carry out the first step in determining the derivative of the function. *Do not simplify any further.* (2 points each)

1. $y = 40x^3 - 39x^2 + 20x + 1$

2. $f(x) = \cos(3x^2 + 1)$

3. $r(t) = \dfrac{e^t + 1}{\tan(t)}$

4. $y = \sin(2x - 3)\cos(4x)$

5. $g(x) = 2^{\sin(x^3)}$

For questions 6 through 8, determine each definite or indefinite integral. Show appropriate justification for your responses. (3 points each)

6. $\int_1^9 x^{-\frac{1}{2}}\,dx$ ____________________

7. $\int_1^2 \dfrac{w^4 + w^2}{w^3}\,dw$ ____________________

8. $\int \left(2^t + \cos(2t) - 3e^t\right)dt$ ____________________

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9. Use implicit differentiation to determine the derivative of $y^2+4xy-x^3=y$. (5 points)

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10. Determine the equation of the line tangent to $y=x^3+3x^2+2$ at its point of inflection. Write your equation in the form $y=mx+b$. Show appropriate justification for your solution. (5 points)

____

Determine each of the following limits. Provide appropriate justification or explanation for each result. If a limit does not exist, explain why it does not. (2 points each)

11. $\lim_{x\to 1} \frac{x^2-4x+3}{x-1}$

____

12. $\lim_{n\to\infty} \frac{4n^2}{n^2+1000n}$

____

13. $\lim_{t\to 0} \frac{\cos(t)-1}{2t^2}$

____

14. The position of a particle moving along a straight line at any time t is given by the function $s(t)=t^2+4t+4$. Determine the acceleration of the particle at time $t=4$.Show appropriate justification for your response. (2 points)

____

15. Use logarithmic differentiation to determine the derivative of $y=x^{\sin(x)}$. (4 points)

____

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Part II: Calculators May Be Used

16. Use <u>the limit definition of derivative</u> to show that the derivative of $f(x)=x^2+3x$ is $f'(x)=2x+3$.
(5 points)

Here is the graph of a function $y = f(x)$. Use it to answer questions 17 through 20. No justification is required. (2 points each)

17. State a value c such that f is continuous at c but f is not differentiable at c.

____

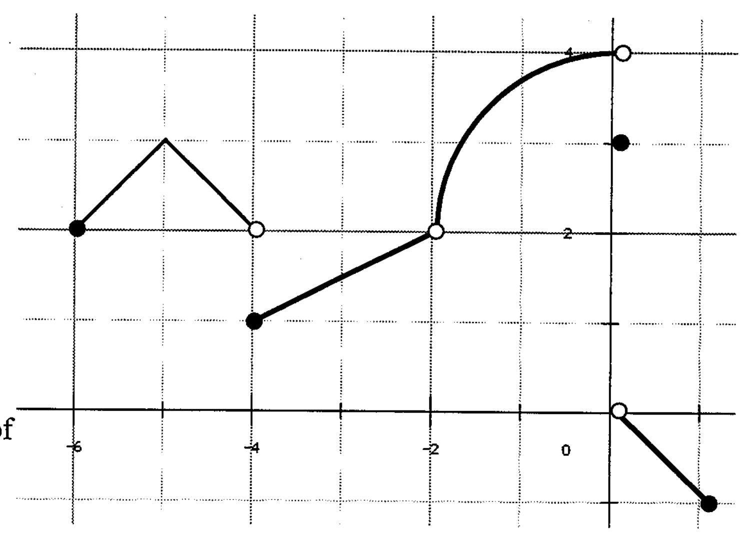
18. Determine $f'(-3)$. ____________

19. For what value c is $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$?

____

20. On the domain $-6 \leq x \leq 1$, for what value(s) of x is the function undefined?

____



For problems 21 through 25 (continued on next page), refer to the following problem situation.Show appropriate justification for your responses. (3 points each)

Water is running into a cylindrical drum at the rate of $w'(t)$ gallons per minute.

21. Explain the meaning of $w'(6)=8$ for this situation.

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22. Explain the meaning of $\int_0^{10} w'(t)\,dt$ for this situation.

Suppose we also know that the drum was empty at time $t = 0$ and that for $t > 0$, $w'(t) = \frac{7}{3}t - \frac{1}{6}t^2$.

23. How much water was added to the drum during the first 2 minutes? ____________________

24. At what time is water running into the tank at the fastest rate? ____________________

25. If the tank holds 55 gallons of water, how long does it take to fill the tank? ____________________

Use the graph here for questions 26 through 28. Note that this is a graph of $g'(x)$, the <u>derivative</u> of some function g. (2 points each)

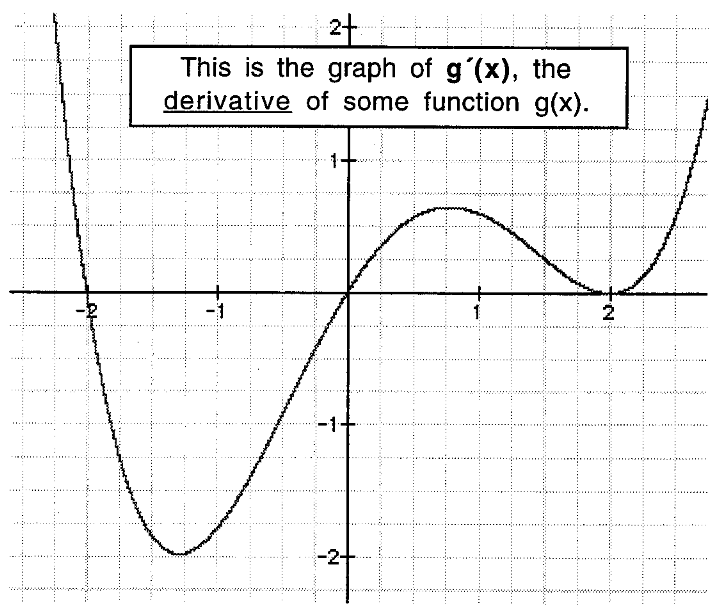
26. For what values of x, $-5/2 < x < 5/2$, does g have a local minimum? Explain.

____

27. For what values of x, $-5/2 < x < 5/2$, is g concave up? Explain.

____

28. Describe the behavior of the function $y = g(x)$ around $x = 2$.



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For questions 29 through 32, assume that a rat is moving in a straight hollow tube with velocity $v(t)=t^3-10t^2+27t-18$ cm/sec. Show appropriate justification for your responses. (11 points total)

29. Determine the velocity of the rat at time $t=2$ seconds. ____________

30. Determine the time intervals over which the rat is moving to the left. ____________

31. Determine the total distance traveled by the rat for $0 \le t \le 3$. ____________

32. If we know that at time $t=0$ the rat is at the origin (i.e., its position is 0), what is its position at time $t=6$?

____

For questions 33 through 35, refer to the following situation. Show appropriate justification for your responses. (8 points total)

The volume of a cone ($V=\frac{1}{3}\pi r^2 h$) is increasing at the rate of 28π cubic units per second. At the instant when the radius r of the cone is 3 units, its volume is 12π cubic units and the radius is increasing at ½ unit per second.

33. At the instant when the radius of the cone is 3 units, what is the height of the cone? ____________

34. At the instant when the radius of the cone is 3 units, what is the rate of change of the area of the base?

____

35. At the instant when the radius of the cone is 3 units, what is the rate of change of its height h?

____

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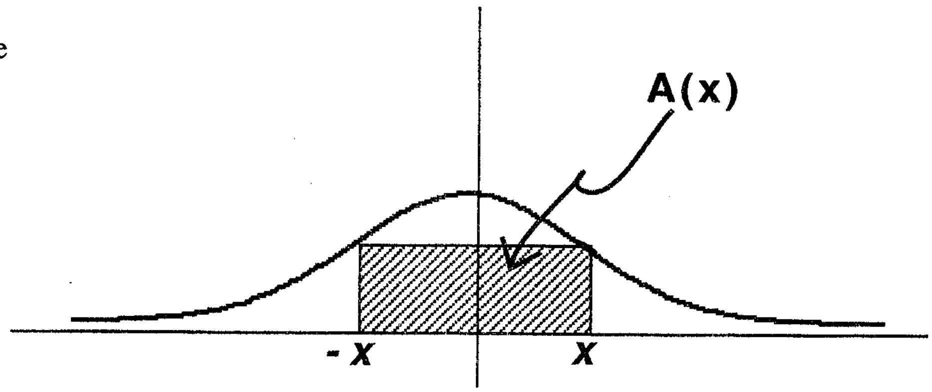
For questions 36 and 37, refer to the following situation. Show appropriate justification for your responses. (6 points total)

Let $A(x)$ be the area of the rectangle inscribed under the curve $y = e^{-2x^2}$ with vertices on the x–axis at $(-x,0)$ and $(x,0)$, $x \geq 0$, as shown in the figure above.



36. Calculate $A(1)$. __________

37. Determine the maximum value of $A(x)$. __________

Suppose that during a New Year's Day storm, snow starts falling at 8 am and falls throughout the day at the rate $f(t) = \sin\left(\frac{\pi x}{12}\right)$. Use this information to respond to problems 38 through 40. Show appropriate justification for your responses. (3 points each)

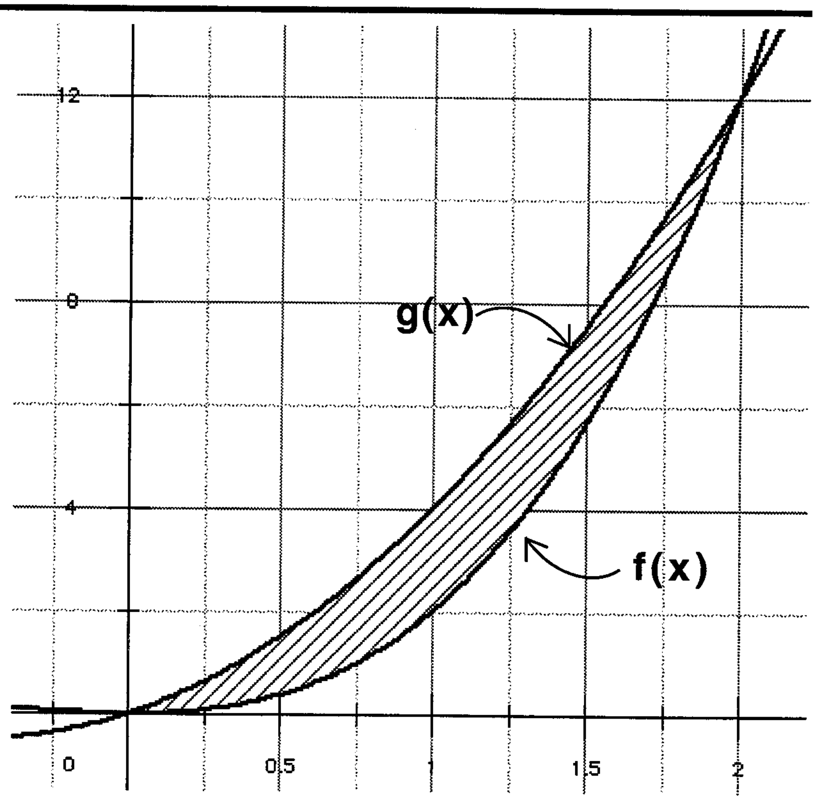
38. For how many hours did the snow fall? __________

39. At what time of day was snow falling the heaviest? __________

40. What was the total accumulation of snow from this storm? __________

41. Suppose we know that $f(x)=\int_0^x \frac{t^2-4}{1+\cos^2(t)}\,dt$. Determine all local maximum and minimum values of $f(x)$. Show appropriate justification for your response. (4 points)

42. The plot here shows first quadrant graphs of $f(x)=x^3+x^2$ and $g(x)=2x^2+2x$. Determine the area of shaded region shown here. Show appropriate justification for your response. (4 points)



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For questions 43 through 46, consider the function $f(x) = \ln(x)$ on the interval $1 \le x \le 5$ and various Riemann sums used to approximate $\int_1^5 f(x)dx$. Show appropriate justification for your responses. (10 points total)

43. If $n = 8$ subdivisions are to be used in a Riemann sum approximation, what is the value of x_6?

44. If $\Delta x = 1/50$ for the Riemann sum approximation, how many subdivisions are there?

45. If $\mathbf{R}_{25}$, $\mathbf{L}_{25}$, and $\mathbf{M}_{25}$ represent Riemann sum approximations using 25 right, left, and midpoint sample points, respectively, order the symbols $\mathbf{R}_{25}$, $\mathbf{L}_{25}$, and $\mathbf{M}_{25}$ from least to greatest. Which of these three estimates will be closest to the actual value of the definite integral?

46. In the Riemann sum $\sum_{i=1}^{n} \Delta x \cdot \ln(1 + i \cdot \Delta x)$, where $\Delta x = \frac{5-1}{n}$, what is the result of evaluating the limit of that sum as n approaches ∞ ?

47. For the function $f(x) = 2x^3 - 3ax^2 - 18x + 60$, where a is a real number, determine the value of a that results in $f(x)$ reaching a local maximum when $x = -1$. Show appropriate justification for your response. (4 points)