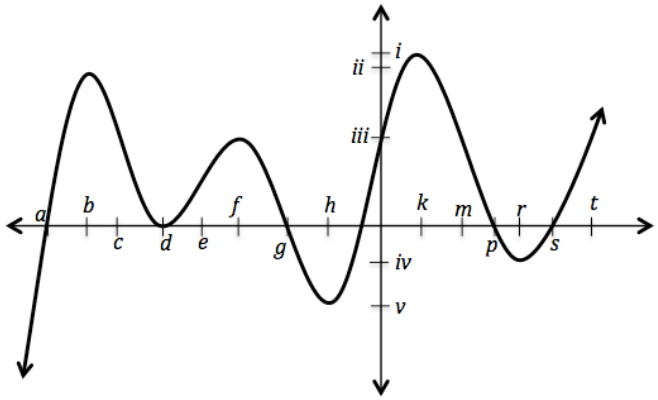
MAT 145

Semester Exam Part II — Name _____

100 points (Part II: 50 points) — Calculator Used _____

Impact on Course Grade: approximately 25% — Score _____

20. The graph of a polynomial function $y = w(x)$ is shown here. Use it to determine responses to questions (a) through (e). **Respond with equations,** such as ***x = m*** or ***y = ii***, **ordered pairs (*x*,*y*)**, **inequalities in *x* or *y***, or **integer values**. (2 pt each)



(a) Use an ***ordered pair*** to state the location of **one first-quadrant local maximum** of *w*.

_____

(b) Use an ***ordered pair*** to state the ***y-intercept*** of the graph of *w*.

_____

(c) State one ***positive x-axis interval*** over which *w* is **increasing**.

_____

(d) State one ***negative x-axis interval*** over which *w* is **concave down**.

_____

(e) Use an ***ordered pair*** to state the point of inflection with the smallest value of *w*.

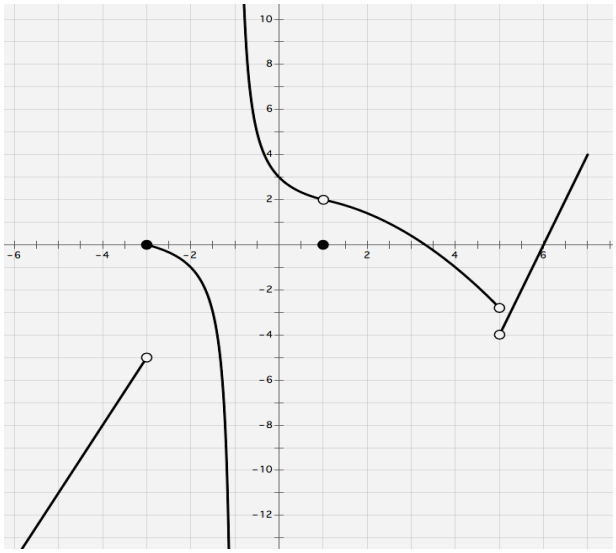
_____

21. Use the graph of the function $y = f(x)$ shown here as you respond to (a)-(e). (1 pt each)

(a) $\lim_{x\to 6} f(x)$ _____ (b) $\lim_{x\to 1} f(x)$ _____

(c) $\lim_{x\to -3^+} f(x)$ _____ (d) $\lim_{x\to -1^+} f(x)$ _____

(e) $f'(-4)$ _____



22-(i)

(a) Consider the statement:

If $f''(1) = 0$, then $(1, f(1))$ is an inflection point of the curve $y = f(x)$.

Which of the following phrases best describes this statement? ***Circle one response.*** (2 pts)

always true *sometimes true* *never true*

(b) Consider the statement:

If $f'(c) = 0$, then f has a ***local maximum*** or a ***local minimum*** at c.

Which of the following phrases best describes this statement? ***Circle one response.*** (2 pts)

always true *sometimes true* *never true*

(c) Consider the statement:

If a function f is continuous and has a local minimum at $x = c$, then $f'(c) = 0$.

Which of the following phrases best describes this statement? ***Circle one response.*** (2 pts)

always true *sometimes true* *never true*

(d) Consider the statement:

If $f'(x) < 0$ for $6 < x < 10$, then f is decreasing on (6,10).

Which of the following phrases best describes this statement? ***Circle one response.*** (2 pts)

always true *sometimes true* *never true*

22-(ii) From among statements (a) through (d), choose **<u>one</u>** that is ***not always true***. In the space here, (i) ***circle*** the letter of your chosen statement, (ii) ***provide one counterexample*** to show that the statement is not always true, and (iii) ***briefly explain*** why your counterexample shows that the statement is ***not always true***. (2 pts)

(i) (a) (b) (c) (d) (circle one)

(ii) counterexample: (iii) explain:

23-(i) For a function $y = f(x)$, its ***derivative*** is $f'(x) = (x-3)(x+2)^2(x-1)$. Determine the ***location*** of all ***local minimum values*** of f, if any exist. Show appropriate evidence and explanation. (2 pts)

_____

23-(ii) For a function $g(x)$, its ***first derivative*** is $g'(x) = \frac{2}{3}x^3 - 2x^2 - 16x + 1$. Determine ***all x-axis intervals***, if any exist, for which *g* is ***concave up***. Show appropriate evidence and explanation. (3 pts)

_____

24. Choose ONE of the three optimization problems described below. Solve the problem, showing ***complete evidence and calculus justification***. Include a drawing or a graph to represent the situation. (5 pts)

A. Determine all ordered pairs on the curve $y = \frac{1}{2}x^2$ that are closest to the point (6,0).

B. A segment through the point (2,7) has an endpoint A = (0,*y*) on the positive *y* axis and another endpoint B = (*x*,0) on the positive *x* axis. If the origin is labeled point P, determine the ordered pairs A and B so that triangle ABP has the smallest area possible.

C. A rectangular field is to be fenced. If we represent the field as rectangle ABCD, the cost of the fence for side AB is $10 per foot, the cost of the fence for side CD is $10 per foot, the cost of the fence for side BC is $3 per foot, and the cost of the fence for side DA is $7 per foot. We have $700 available to pay for fence materials. Determine the dimensions of the rectangle ABCD that will fence in the largest area for $700.

25. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which expression here shows the ***change in altitude*** of the balloon during the time the balloon's altitude is ***decreasing***? Circle one. (2 pts)

(A) $\int_{1.572}^{3.514} r(t)\,dt$ | (B) $\int_{0}^{8} r(t)\,dt$ | (C) $\int_{0}^{2.667} r(t)\,dt$ | (D) $\int_{1.572}^{3.514} r'(t)\,dt$ | (E) $\int_{0}^{2.667} r'(t)\,dt$

Explain how you determined your response. (3 pts)

26. Determine an equation of the line tangent to $y = x^4 - 2x^3 + x + 2$ at $x = -1$. Write your equation in the form $y = mx + b$. Show appropriate justification for your solution. (5 pts)

_____

27. Use a Riemann sum to approximate the area under the curve $y = 4 - x^2$ on the interval $1 \le x \le 2$. Use $n = 4$ subdivisions and use left-hand endpoints. Show a sketch of the function and include the rectangles in your drawing. Your numerical response should be an ***exact value expressed as a common fraction***. (5 pts)

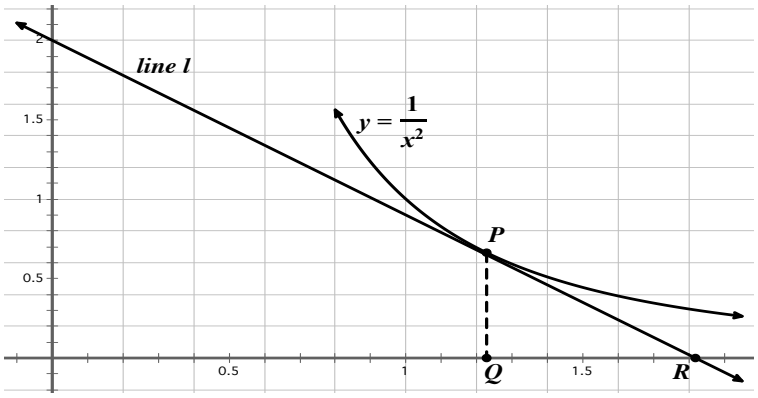
_____

Name _____

BONUS #3

A city was founded in 1900 with an initial population of 400 people. The rate of change of the city's population is modeled by the equation $P'(t) = \frac{1}{4}\sqrt{\frac{3}{10}x+1}$ with ***P*** the population in ***hundreds of people*** and ***t*** measured in **years since 1900**. Use this information to determine an accurate estimate for the city's population in 1950. (3 pts)

BONUS #4



In the figure here, line l is tangent to the graph of $y = \frac{1}{x^2}$ at point P. The coordinates of point P are $\left(w, \frac{1}{w^2}\right)$ with $w > 0$. Point Q has coordinates $(w,0)$. Line l intersects the x-axis at point R. The coordinates of point R are $(k,0)$.

(i) Determine the value of k when $w = 3$. (1 pt)

(ii) For all $w > 0$, represent k in terms of w. (2 pts)

(iii) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time t? (3 pts)

(iv) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of triangle PQR with respect to time t? Determine whether this area is increasing or decreasing at this instant. (4 pts)

Calculus I
MAT 145
Semester Exam

Total Points:	100
Impact of Exam on Semester Grade:	Approximately 25%

Evaluation Criteria

Unless otherwise directed, state any numerical solutions as exact values in rational expressions reduced to lowest terms. If approximations are required, express as a decimal value rounded accurately to the nearest thousandth of a unit, or as requested.

Part I: No Calculators **50 points**

Questions 1 through 10
2 points each with no partial credit. No need to show work on these.

Questions 11 through 15
2 points each. Partial credit is possible. Show all steps leading to your solutions. Be clear, complete, and accurate. Simplify if possible.

Questions 16 through 19
5 points each. Partial credit is possible. Show all steps leading to your solutions. Be clear, complete, and accurate. Simplify if possible.

16) (i) 2 pts: correct response; (ii) 3 pts: correct implicit differentiation (2 pts) and correct determination of exact value of the slope (1 pt)
17) 1 pt for each correct response
18) (i) 4 pts: correct exact numerical values; (ii) 1 pt: correct symbolic representation for G
19) 5 pts: include initial statement of the definition of derivative, well-organized progression of steps of simplification, correct use of symbols throughout, and clearly labeled result

Bonus #1 (8 pts), **Bonus #2** (5 pts): Provide complete and accurate evidence and response.

Part II: Calculators Allowed **50 points**

Partial credit is possible. Show all steps leading to your solutions. Be clear, complete, and accurate.

20) 1 pt each, using correct format (ordered pair or inequality)
21) 1 pt each, correct response
22) (i) 8 pts: correct response choices; (ii) 2 pts: accurate counterexample with appropriate explanation
23) (i) 2 pts: accurate statement of locations of local min values, with evidence; (c) 3 pts: accurate statement of x-axis intervals, with evidenc
24) 5 pts: 1 pt for required sketch, drawing, or graph; 1 pt for statement of function to be optimized; 1 pt for use of and reference to a constraint; 2 pts for correct solution with calculus evidence
25) 2 pts: correct choice; 3 pts correct, clear, accurate explanation
26) 5 pts: correct tangent-line equation, with required evidence, written in appropriate format
27) 5 pts: correct calculation of exact value of the Riemann sum, with function sketch showing rectangles

Bonus #3 (3 pts), **Bonus #4** (7 pts): Provide complete and accurate evidence and response.

Name _____ email _____ text #_____

Gateway:_____ /20 No Calc: _____ /30 Calc: _____ /50 Bonus:_____ TOTAL: _____ /100