MAT 145
Spring 2013
Semester Exam

Test-taking guidelines: *This test is to be completed individually during the final-exam period. For the first part of the exam, you may NOT use your calculator, textbook, class notes, or any other materials. For the second part of the exam, you may use your calculator, but you may not use your textbook, class notes, or any other materials. You may not discuss the content of the test with anyone other than your instructor.*

Certification: In completing this in-class exam, I have followed the test-taking directions above and have seen no dishonest work.

Name (printed)	*UID #*	*Signature*	*Date*

Part I: No Calculators

Problems	Points Available	Points Earned
1-5	10	
6-10	10	
11-13	6	
14-15	4	
Total	30	

Part II: Calculators Allowed

Problems	Points Available	Points Earned
16-20	20	
21-24	15	
25-30	18	
31-34	9	
35-39	8	
Bonus!	8	
Total	70	

Do your best work! Have a great summer!

Semester Exam Name ____________________

Part I: Calculators May Not Be Used

General Directions: *Show all work. Use appropriate notation. When explanations are required, write in full sentences and be sure that your reasoning is clear.*

For each function in questions 1 through 5, determine the derivative of the function. *Show your first two steps and then do not simplify any further.* (2 points each)

1. $y = 6x^5 - 3x^8 + 2e^3$ ____________________

2. $f(x) = \cos(\tan(x))$ ____________________

3. $r(x) = \dfrac{x^2}{1-x^2}$ ____________________

4. $y = (4e^x - 3)(x^5)$ ____________________

5. $g(x) = (3\ln x)^3$ ____________________

For questions 6 through 8, determine each definite or indefinite integral. Show appropriate justification for your responses. (2 points each)

6. $\int_{1}^{2} x^3 \, dx$ ________________________

7. $\int_{4}^{5} \frac{t^2 - 4}{t + 2} \, dt$ ________________________

8. $\int \left(\sin x + \frac{1}{x} \right) dx$ ________________________

9. Use implicit differentiation to determine the derivative of $x^2 y - 3y = x$. (2 points)

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10. Determine the equation of the line tangent to $y = 3x^3 + x^2 - 2x + 4$ at its y-intercept. Write your equation in the form $y = mx + b$. Show appropriate justification for your solution. (2 pts)

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Determine each of the following limits. Provide appropriate justification or explanation for each result. If a limit does not exist, explain why it does not. (2 points each)

11. $\lim_{x \to 2} 3x^2 - 2x - 1$ ____________________

12. $\lim_{t \to \infty} \frac{6t^5}{5t^5 - 67t^4}$ ____________________

13. $\lim_{x \to 0} \left(\frac{\sin(2x)}{3x} \right)$ ____________________

14. The position, measured in ***meters*** from the origin, of a particle moving along a straight line at any time *t*, in ***minutes***, is given by the function $s(t) = t^5 - 4t^3 + 3$. Determine the acceleration of the particle at time $t = 1$ minute. Show appropriate justification for your response. ***Be sure to show appropriate units!*** (2 points)

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15. Use <u>logarithmic differentiation</u> to determine the derivative of $y = x^{\sin(x)}$. (2 points)

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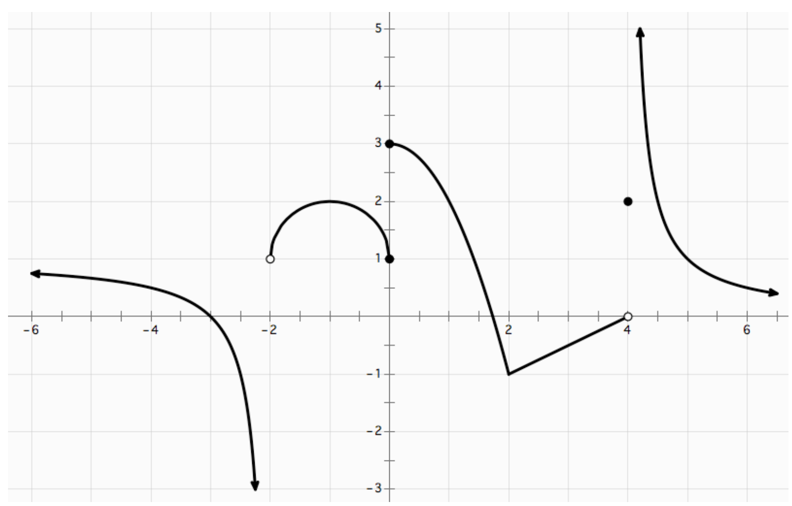
Part II: Calculators May Be Used

General Directions: *Show all work. Use appropriate notation. When explanations are required, write in full sentences in order to be sure that your reasoning is clear. If you use a calculator to perform computations, specify both the calculator input and output.*

16. Use the limit definition of derivative to determine the derivative of $f(x)=\sqrt{5x}$. To earn any credit for your solution, you must ***show appropriate algebraic evidence*** leading from the definition of derivative to a final simplified derivative. Take care in using appropriate labels as you proceed. (6 points)



Shown here is the graph of the relationship *y = f(x)*. Use the graph to answer questions 17 through 23. No justification is required. (2 points each)

17. State all values of *c* for which *f* is continuous at *c* but not differentiable at *c*. ________________

18. Determine $f'(3)$. ________________

19. State all values of *k* for which $\lim_{x\to k^-} f(x) \neq \lim_{x\to k^+} f(x)$. ________________

20. On the interval $-6 \le x \le 6$, state all values of *x* for which the function is undefined. ___________

21. For $x > 4$, is *f* concave up or concave down? Circle one: Concave Up Concave Down

22. State equations, in the form $y = d$, for all horizontal asymptotes: __________________________

23. On the interval $-6 \le x \le 6$, there is exactly one aspect of the graphed relationship that prevents *y = f(x)* from being the graph of a ***function***. Identify and describe this aspect.

For questions 24 through 26, consider the following situation. Show appropriate justification for your responses. ***Be sure to show appropriate units!*** (10 points total: 2, 4, 4)

A particle is moving up and down (down: negative, up: positive) on the *y*-axis. The particle's position, measured in centimeters from the origin (with coordinate 0), at any time *t*, in seconds, is given by the function $s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t - 6$, $t \geq 0$.

24. Determine the position of the particle relative to the origin at time $t = 3$ seconds.____________

25. Determine all time intervals, $t \geq 0$, over which the particle is moving up. __________________

26. Determine the *total distance traveled* by the particle from time $t = 1$ to time $t = 7$.____________

27. Calculate the first-quadrant area under the curve $y = \sin(x)$ on the interval $0 \leq x \leq \pi$. Make a sketch to show the region, show a definite integral used to calculate the area, and report the exact area. (5 pts.)

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Use the following information to respond to problems 28 through 30. Show appropriate justification for your responses. ***Be sure to show appropriate units!*** (3 points each)

Beginning at 12 noon on a Monday in May, rain started falling at the rate
$r(t) = \frac{1}{2} - \frac{1}{98}(x-7)^2$ inches per hour.

28. How many hours passed until the rain first stopped falling? ____________________________

29. At what time of day on Monday was rain falling the heaviest? ____________________________

30. What was the total accumulation of rain from this storm, that is, the amount of rain that fell from the time the rain started to when it first stopped falling? Express as a decimal value to the nearest thousandth of an inch.

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For questions 31 through 33, refer to the following situation. Show appropriate justification for your responses. (9 points total [2,3,4])

Farmer Fred Flannagan wants to enclose a rectangular region for his cattle. He will use one exterior wall of an existing barn as one side of the rectangle and will build onto the barn, using fence, the three additional sides of the rectangle. Fred will use exactly 240 meters of fence. He needs to know the dimensions of the rectangle he should build that will enclose the greatest area. We will use $A(x)$ to represent the area of the rectangle whose side length, along the barn, is x meters long.

31. Calculate $A(30)$. ____________________________

32. Create a function $A(x)$ to represent the area of the rectangle. ____________________________

33. Use calculus to determine the maximum area Fred can enclose under these circumstances.

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34. Use a Riemann sum to approximate the area under the curve $y = x^2$ on the interval $0 \le x \le 2$. Use $n = 4$ subdivisions and use right-hand endpoints Show a sketch of the function and include the rectangles in your drawing. (6 points)

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35. State a function for which $f(x) = f'(x)$. You do not need to justify your response. (3 points)

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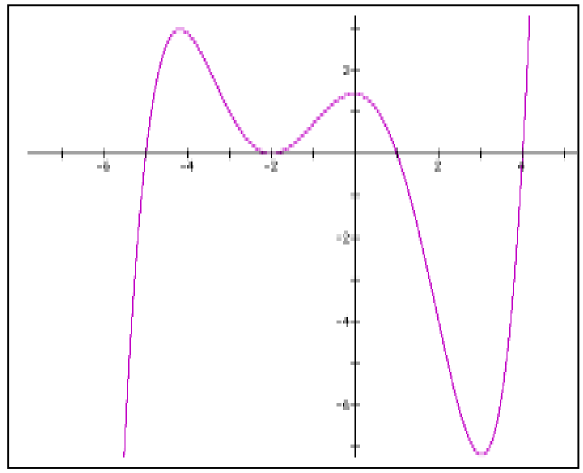
Use the graph here for questions 36 through 39. The graph shown here is a graph of $\boldsymbol{g'(x)}$, the ***derivative*** of some function g. Note that each tick mark on the axes represents one unit.
(2 points each)



36. Estimate all values of x, $-6 \le x \le 4$, for which g has a local minimum? Explain how you know.

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37. Estimate all intervals of x values, $x \in \Re$, for which g is concave down. Explain. ________

38. Describe and explain the behavior of the function $y = g(x)$ near $x = -5$.

39. For what value of x on $-4 \le x \le 4$ does the function $y = g(x)$ take on its global minimum? Explain.

BONUS! State and correctly spell the first and last name of each of the two individuals credited with discovering Calculus. (8 points)

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