SOLUTION GUIDE

MAT 146

Quiz #8: Sample (Series Convergence) Name _____

10 points

Impact on Course Grade: approximately 2% Score _____

Show appropriate calculus evidence to fully support your responses.
Show exact values unless otherwise requested.

For each infinite series, state whether the series is convergent or divergent. If a series is convergent, state the value of the series. No evidence nor justification is required for these.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ (Convergent) Divergent (Circle one.) Sum (if convergent): -0.9015427

2. $\sum_{n=1}^{\infty} \frac{7^{n+1}}{5^{n-1}}$ Convergent (Divergent) (Circle one.) Sum (if convergent): X

3. $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^n}$ (Convergent) Divergent (Circle one.) Sum (if convergent): ≈ 0.71342836

4. Provide a proof for whether $\sum_{n=1}^{\infty} \frac{2n^2+5}{3n^3}$ is a divergent series or a convergent series.

Explain and justify your response, including use of one or more convergence tests.

We will compare $a_n = \frac{2n^2+5}{3n^3}$ to $b_n = \frac{1}{n}$; $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{2}{3}$;

The series $\sum_{n=1}^{\infty} b_n$ is a positive series, as is $\sum_{n=1}^{\infty} b_n$ so we can use the LIMIT COMPARISON TEST. Because $\sum_{n=1}^{\infty} b_n$ diverges, and we have $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{2}{3}$, a positive finite value, we know, by the LIMIT COMPARISON TEST, that $\sum_{n=1}^{\infty} \frac{2n^2+5}{3n^3}$ also DIVERGES.

BONUS! Lamar knows that $\sum_{n=1}^{\infty} \frac{4}{4n^2+12n+5}$ converges, but he doesn't know the sum. On the back side of this page, show how to determine the exact value of this series.

$$\sum_{n=1}^{\infty} \frac{4}{4n^2+12n+5} = \sum_{n=1}^{\infty} \left(\frac{1}{2n+1} - \frac{1}{2n+5}\right)$$

$$= \left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{5} - \frac{1}{9}\right) + \left(\frac{1}{7} - \frac{1}{11}\right) +$$

$$\left(\frac{1}{9} - \frac{1}{13}\right) + \cdots$$

(=0, =0, =0, =0)

$$= \left(\frac{1}{3} + \frac{1}{5}\right)$$

$$= \frac{5+3}{15} = \frac{8}{15}$$