

SOLUTION GUIDE

MAT 146

Quiz #8: Sample (Series Convergence)

Name _____

10 points

Impact on Course Grade: approximately 2%

Score _____

Show appropriate calculus evidence to fully support your responses.
Show exact values unless otherwise requested.

For each infinite series, state whether the series is convergent or divergent. If a series is convergent, state the value of the series. No evidence nor justification is required for these.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ Convergent Divergent Sum (if convergent): -0.9015427
(Circle one.)

2. $\sum_{n=1}^{\infty} \frac{7^{n+1}}{5^{n-1}}$ Convergent Divergent Sum (if convergent): X
(Circle one.)

3. $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^n}$ Convergent Divergent Sum (if convergent): ≈ 0.71342836
(Circle one.)

4. Provide a proof for whether $\sum_{n=1}^{\infty} \frac{2n^2+5}{3n^3}$ is a divergent series or a convergent series.

Explain and justify your response, including use of one or more convergence tests.

We will compare $a_n = \frac{2n^2+5}{3n^3}$ to $b_n = \frac{1}{n}$; $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{2}{3}$;

The series $\sum_{n=1}^{\infty} a_n$ is a positive series, as is $\sum_{n=1}^{\infty} b_n$ so we can use the LIMIT COMPARISON TEST. Because $\sum_{n=1}^{\infty} b_n$ diverges, and we have $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{2}{3}$, a positive finite value, we know, by the LIMIT COMPARISON TEST, that $\sum_{n=1}^{\infty} \frac{2n^2+5}{3n^3}$ also diverges.

BONUS! Lamar knows that $\sum_{n=1}^{\infty} \frac{4}{4n^2+12n+5}$ converges, but he doesn't know the sum. On the back side of this page, show how to determine the exact value of this series.

$$\sum_{n=1}^{\infty} \frac{4}{4n^2+12n+5} = \sum_{n=1}^{\infty} \left(\frac{1}{2n+1} - \frac{1}{2n+5} \right)$$

$$= \left(\frac{1}{3} - \frac{1}{7} \right) + \left(\frac{1}{5} - \frac{1}{9} \right) + \left(\frac{1}{7} - \frac{1}{11} \right) + \left(\frac{1}{9} - \frac{1}{13} \right) + \dots$$

Diagram illustrating the telescoping nature of the series with cancellation arrows labeled = 0:

- Arrow from $\frac{1}{7}$ in the first term to $\frac{1}{7}$ in the third term, labeled = 0.
- Arrow from $\frac{1}{9}$ in the second term to $\frac{1}{9}$ in the fourth term, labeled = 0.
- Arrow from $\frac{1}{11}$ in the third term to $\frac{1}{11}$ in the next term, labeled = 0.
- Arrow from $\frac{1}{13}$ in the fourth term to $\frac{1}{13}$ in the next term, labeled = 0.

$$= \left(\frac{1}{3} + \frac{1}{5} \right)$$

$$= \frac{5+3}{15} = \frac{8}{15}$$