Topic: The Chain Rule

When taking the derivative of a composition of functions, the chain rule must be used. Function composition is when one function is applied, then its output is fed into another function; it is written as $f(g(x))$, where the function g is applied first and f is applied second. An example of function composition is $\sin(\cos(x))$; first cos is applied to x, and the output of cos is then fed into sin. In this case, we let $g(x) = \cos(x)$ and $f(x) = \sin(x)$. Another example is $(x^2 - 1)^5$; in this case, $g(x) = x^2 - 1$ and $f(x) = x^5$. Identifying what you should use as f and g is essential to properly applying the chain rule.

The chain rule says: $\boxed{(f(g(x)))' = f'(g(x))g'(x)}$

Examples:

1. Find the derivative of $(3x^7 - 2x^4 + 4)^{22}$.

We set $g(x) = 3x^7 - 2x^4 + 4$ since this is applied first. The result of g is raised to the 22nd power, so we set $f(x) = x^{22}$. To apply the chain rule, we differentiate $f'(x) = 22x^{21}$ and $g'(x) = 21x^6 - 8x^3$. Our final result is

$$f'(g(x))g'(x) = 22(3x^7 - 2x^4 + 4)^{21}(21x^6 - 8x^3)$$

2. Find the derivative of $\cos(x^5)$.

We set $g(x) = x^5$ and $f(x) = \cos(x)$. Then, we differentiate $f'(x) = -\sin(x)$ and $g'(x) = 5x^4$. Our final result is

$$f'(g(x))g'(x) = -\sin(x^5) \cdot 5x^4$$

3. Find the derivative of $(\cos x + 5\sin x)^8$.

We set $g(x) = \cos x + 5\sin x$ and $f(x) = x^8$. Then, we differentiate $f'(x) = 8x^7$ and $g'(x) = -\sin x + 5\cos x$. Our final result is

$$f'(g(x))g'(x) = 8(\cos x + 5\sin x)^7(-\sin x + 5\cos x)$$

4. Find the derivative of 3^{4-x^2}.

We set $g(x) = 4 - x^2$ and $f(x) = 3^x$. Then, we differentiate $f'(x) = \ln(3) \cdot 3^x$ and $g'(x) = -2x$. Our final result is

$$\ln(3) \cdot 3^{4-x^2} \cdot (-2x) = -2x\ln(3) \cdot 3^{4-x^2}$$