Topic: Fractions

Background: Assume in all rules below that no denominator is zero.

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ Add numerator when denominator are equal

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ Find a common denominator

$\frac{a}{c} . \frac{b}{d} = \frac{ab}{cd}$ Multiply numerator and denominator in a product

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$ To divide by a fraction multiply by its reciprocal

$\frac{\frac{a}{b}}{c} = \frac{a}{bc}$ Same as division by $\frac{c}{1}$

$\frac{a}{\frac{b}{c}} = \frac{ac}{b}$ To divide by a fraction multiply by its reciprocal

$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

Illustrative Examples:

(1) Simplify the following expression. Assume any factors you cancel are not zero.

$$\frac{\frac{7}{k+1}-1}{\frac{2}{k+1}-1}.$$

Solution:

$$\begin{aligned}
\frac{\frac{7}{k+1}-1}{\frac{2}{k+1}-1} &= \frac{\frac{7-(k+1)}{(k+1)}}{\frac{2-(k+1)}{(k+1)}} \\
&= \frac{(6-k)}{(k+1)}.\frac{(k+1)}{(1-k)} \quad \text{(Cancel } (k+1) \text{ from numerator and denominator)} \\
&= \frac{(6-k)}{(1-k)}
\end{aligned}$$

(2) Simplify and write the following expression as a single fraction.

$$3+\frac{x}{5}+\frac{2}{x}+\frac{7}{x^2}$$

Solution:

$$\begin{aligned}
3+\frac{x}{5}+\frac{2}{x}+\frac{7}{x^2} &= \frac{(3).(5x^2)+(x).(x^2)+(2).(5x)+(7).(5)}{5x^2} \\
&= \frac{15x^2+x^3+10x+35}{5x^2}
\end{aligned}$$

(3) Simplify and write the following expression as a single fraction. Assume any factors you cancel are not zero.

$$\frac{a+b}{a^{-2}+b^{-2}}$$

Solution:

$$
\begin{aligned}
\frac{a+b}{a^{-2}-b^{-2}} &= \frac{(a+b)}{\frac{1}{a^2}-\frac{1}{b^2}} \\
&= \frac{(a+b)}{\frac{b^2-a^2}{a^2b^2}} \\
&= \frac{(a+b)a^2b^2}{b^2-a^2} \\
&= \frac{(a+b)a^2b^2}{(b+a)(b-a)} \quad \text{(factorize } b^2-a^2\text{)} \\
&= \frac{a^2b^2}{(b-a)} \quad \text{(Cancel } (a+b) \text{ from numerator and denominator.)}
\end{aligned}
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