Topic: Numerical fractions

To add fractions when the denominators are equal, simply add the numerators: $\boxed{\dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}}$

To add fractions when the denominators are not equal, you must first find a common denominator. One way to get a common denominator is to multiply the denominators: $\boxed{\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad}{bd} + \dfrac{bc}{bd} = \dfrac{ad+bc}{bd}}$. Note that this will not always give the least common denominator, a topic we will return to.

The rules for subtraction are analogous to those for addition.

To multiply fractions, simply multiply numerators and multiply denominators: $\boxed{\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}}$

To divide by a fraction, multiply by its reciprocal: $\boxed{\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}}$

A negative sign has the same effect whether placed on top, on bottom, or in front of a fraction: $\boxed{-\dfrac{a}{b} = \dfrac{-a}{b} = \dfrac{a}{-b}}$

A fraction is reduced by dividing numerator and denominator by a common factor: $\boxed{\dfrac{a}{b} = \dfrac{a \div c}{b \div c}}$

The value of a fraction is also preserved when multiplying numerator and denominator by the same thing: $\boxed{\dfrac{a}{b} = \dfrac{ac}{bc} \text{ for } c \neq 0}$

Mixed numbers can be converted to improper fractions as follows: $\boxed{a\dfrac{b}{c} = \dfrac{ac+b}{c}}$

Least common denominators: The least common denominator (LCD) of two fractions is given by the least common multiple (LCM) of the denominators of the fractions. The purpose of using least common denominators is to avoid having to reduce the fraction later.

The most reliable way to find the LCM of two numbers is to use prime factorization. If you want the LCM of two numbers a and b, start by finding the prime factorization of each number. If, for example, a has three factors of 2 while b has 5, then the LCM will have 5 (the larger number). More generally, for each prime dividing a or b, find the largest power of the prime dividing a or b, and then assign the given prime with the given largest power as a factor of the LCM. Once this is done for all primes dividing a or b, multiply the result to find the LCM.

For example, suppose we wanted to add $\dfrac{1}{12} + \dfrac{1}{30}$. If we just multiplied the denominators, we would get a denominator of 360. Instead, we look for the LCD by finding the LCM of 12 and 30. The prime factorizations are $12 = 2^2 \cdot 3^1$ and $30 = 2^1 \cdot 3^1 \cdot 5^1$. The highest occurring power of 2 is the second power, while the highest occurring powers of 3 and 5 are both the first power. So our LCM is $2^2 \cdot 3^1 \cdot 5^1 = 60$, much smaller than 360. The 12 is missing one power of 5, so we multiply $\dfrac{1}{12} \cdot \dfrac{5}{5} = \dfrac{5}{60}$. The 30 is missing one power of 2, so we multiply $\frac{1}{30} \cdot \frac{2}{2} = \frac{2}{60}$. Finally, we add: $\frac{1}{12} + \frac{1}{30} = \frac{5}{60} + \frac{2}{60} = \frac{7}{60}$. Note that 7 and 60 have no common factors, so the fraction cannot be reduced. (If we used a denominator of 360, the resulting fraction would have to be reduced.)

Examples

1. Write 0.35 as a common fraction.

Since there are two decimal places, we find that $.35 = \frac{35}{100}$. We can reduce this fraction by dividing top and bottom by 5: $\frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$.

2. Find $\frac{5}{8} - \frac{6}{5}$.

Since 8 and 5 have no factors in common, the LCD will be $8 \cdot 5 = 40$. The first fraction should be multiplied by 5 on top and bottom, while the second fraction should be multiplied by 8 on top and bottom: $\frac{5}{8} \cdot \frac{5}{5} - \frac{6}{5} \cdot \frac{8}{8} = \frac{25}{40} = \frac{48}{40} = \frac{-23}{40}$.