NAME:____________________

Math 407
Spring 2010
Dr. Seelinger

HOMEWORK # 9

1. Let G be a finitely generated abelian group in which no element except 0 has finite order. Prove that G is a free abelian group.

2. Let F be a finitely generated free abelian group of rank n. Prove that for any positive integer m that $F/mF \cong (\mathbb{Z}_m)^n$.

3. Consider the abelian group of rational numbers $\mathbb{Q}$ under addition.

   (a) Prove that $\mathbb{Q}$ is NOT a free abelian group. (HINT: Show that any subset of two or more elements cannot be linearly independent.)

   (b) Find a free abelian group F and a surjective homomorphism $\phi : F \to \mathbb{Q}$.

4. Let $\mathbb{Q}^+$ be the group of positive rational numbers with multiplication as the binary operation.

   (a) Prove that $\mathbb{Q}^+$ is a free abelian group. (HINT: Use the Fundamental Theorem of Arithmetic.)

   (b) Find a set X and an isomorphism $\phi : \bigoplus_{x \in X} \mathbb{Z} \to \mathbb{Q}^+$.

5. Use the Fundamental Theorem of Finitely Generated Abelian Groups to list (up to isomorphism) all abelian groups of order 32 and all abelian groups of order 540.