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<http://smartURL.it/krzysioP> (paper) or <http://smartURL.it/krzysioPe> (electronic)

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Exercise for January 14, 2006

November 1981 Course 110 Examination, Problem No. 38

Suppose X and Y have the joint density function

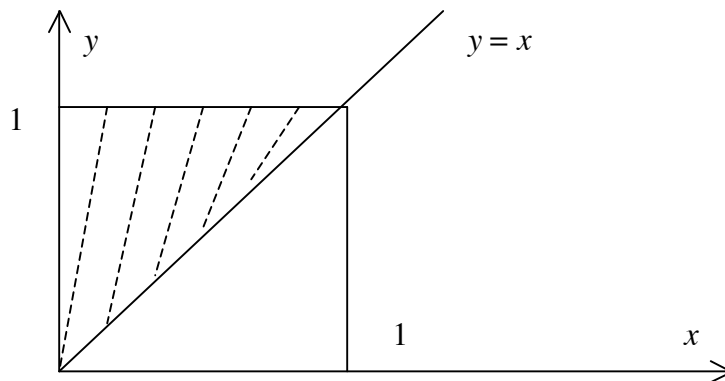
$$f(x,y) = x + \frac{10}{3}y^2$$

for $0 \leq x \leq y \leq 1$. What is $E(XY)$?

- A. $\frac{19}{60}$ B. $\frac{1}{3}$ C. $\frac{11}{30}$ D. $\frac{2}{5}$ E. $\frac{31}{90}$

Solution.

The region where the joint density is positive is shown in the figure below



Therefore,

$$\begin{aligned}
E(XY) &= \int_0^1 \left(\int_x^1 xy \cdot \left(x + \frac{10}{3} y^2 \right) dy \right) dx = \int_0^1 \left(\left(\frac{x^2 y^2}{2} + \frac{5xy^4}{6} \right) \Big|_{y=x}^{y=1} \right) dx = \\
&= \int_0^1 \left(\frac{x^2}{2} + \frac{5x}{6} - \frac{x^4}{2} - \frac{5x^5}{6} \right) dx = \left(\frac{x^3}{6} + \frac{5x^2}{12} - \frac{x^5}{10} - \frac{5x^6}{36} \right) \Big|_{x=0}^{x=1} = \\
&= \frac{1}{6} + \frac{5}{12} - \frac{1}{10} - \frac{5}{36} = \frac{30}{180} + \frac{75}{180} - \frac{18}{180} - \frac{25}{180} = \frac{62}{180} = \frac{31}{90}.
\end{aligned}$$

Answer E.

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