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Exercise for January 7, 2006

Society of Actuaries November 2005 Course M Examination, Problem No. 26
For an insurance:
(i) Losses have density function

$$f_X(x) = \begin{cases} 0.02x, & 0 < x < 10, \\ 0, & \text{elsewhere.} \end{cases}$$

(ii) The insurance has an ordinary deductible of 4 per loss.
(iii) Y^P is the claim payment per payment random variable.
Calculate $E\left(Y^P\right)$.

A. 2.9 B. 3.0 C. 3.2 D. 3.3 E. 3.4

Solution.
Let X be the random variable describing the loss for this insurance. Then $Y^P = \left(X - 4 \middle| X > 4\right)$. Note that since X assumes values between 0 and 10, Y^P only assumes values between 0 and 6. We have, for $0 < y < 6$,

$$\Pr\left(Y^P > y\right) = s_{Y^P}(y) = \Pr\left(X - 4 > y \middle| X > 4\right) = \Pr\left(X > y + 4 \middle| X > 4\right) = \frac{\Pr(X > y + 4)}{\Pr(X > 4)} =$$

$$= \frac{\int_{y+4}^{10} 0.02x\,dx}{\int_{4}^{10} 0.02x\,dx} = \frac{0.01x^2\Big|_{y+4}^{10}}{0.01x^2\Big|_{4}^{10}} = \frac{100 - (y+4)^2}{100 - 16} = \frac{100 - (y+4)^2}{84}.$$

Hence, using the Darth Vader Rule, we obtain

$$E\left(Y^P\right) = \int_0^6 \frac{100 - (y+4)^2}{84}\,dy = \frac{100}{84} \cdot 6 - \left(\frac{(y+4)^3}{3 \cdot 84}\Bigg|_{y=0}^{y=6}\right) =$$

$$= \frac{50}{7} - \left(\frac{250}{63} - \frac{16}{63}\right) = \frac{50}{7} - \frac{26}{7} = \frac{24}{7} \approx 3.42857143.$$

Answer E. You can also do this problem using the basic definition of the expected value, but for that you must first establish the PDF of the random variable Y^P. We have

$$f_{Y_P}(y) = \frac{f_X(y+4)}{\Pr(X>4)} = \frac{0.02\cdot(y+4)}{1-\int_0^4 0.02x\,dx} = \frac{0.02\cdot(y+4)}{1-\left(0.01x^2\Big|_{x=0}^{x=4}\right)} = \frac{0.02\cdot(y+4)}{0.84} = \frac{y+4}{42}.$$

Therefor

$$E\left(Y^P\right) = \int_0^6 y\cdot\frac{y+4}{42}\,dy = \int_0^6\left(\frac{y^2}{42}+\frac{2y}{21}\right)dy = \left(\frac{y^3}{126}+\frac{y^2}{21}\right)\Bigg|_{y=0}^{y=6} = \frac{6^3}{126}+\frac{6^2}{21} = \frac{24}{7} \approx 3.42857143.$$

Answer E, again.