

Krzysztof Ostaszewski: <http://www.krzysio.net>

Author of the "Been There Done That!" manual for Course P/1

<http://smartURL.it/krzysioP> (paper) or <http://smartURL.it/krzysioPe> (electronic)

Instructor of online P/1 seminar: <http://smartURL.it/onlineactuary>

If you find these exercises valuable, please consider buying the manual or attending the seminar, and if you can't, please consider making a donation to the Actuarial Program at Illinois State University: <https://www.math.ilstu.edu/actuary/giving/>

Donations will be used for scholarships for actuarial students. Donations are tax-deductible to the extent allowed by law.

Questions about these exercises? E-mail: krzysio@krzysio.net

Exercise for January 7, 2006

Society of Actuaries November 2005 Course M Examination, Problem No. 26

For an insurance:

(i) Losses have density function

$$f_X(x) = \begin{cases} 0.02x, & 0 < x < 10, \\ 0, & \text{elsewhere.} \end{cases}$$

(ii) The insurance has an ordinary deductible of 4 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $E(Y^P)$.

A. 2.9 B. 3.0 C. 3.2 D. 3.3 E. 3.4

Solution.

Let X be the random variable describing the loss for this insurance. Then

$Y^P = (X - 4 | X > 4)$. Note that since X assumes values between 0 and 10, Y^P only assumes values between 0 and 6. We have, for $0 < y < 6$,

$$\begin{aligned} \Pr(Y^P > y) &= s_{Y^P}(y) = \Pr(X - 4 > y | X > 4) = \Pr(X > y + 4 | X > 4) = \frac{\Pr(X > y + 4)}{\Pr(X > 4)} = \\ &= \frac{\int_{y+4}^{10} 0.02x dx}{\int_4^{10} 0.02x dx} = \frac{0.01x^2 \Big|_{y+4}^{10}}{0.01x^2 \Big|_4^{10}} = \frac{100 - (y+4)^2}{100 - 16} = \frac{100 - (y+4)^2}{84}. \end{aligned}$$

Hence, using the Darth Vader Rule, we obtain

$$\begin{aligned} E(Y^P) &= \int_0^6 \frac{100 - (y+4)^2}{84} dy = \frac{100}{84} \cdot 6 - \left(\frac{(y+4)^3}{3 \cdot 84} \Big|_{y=0}^{y=6} \right) = \\ &= \frac{50}{7} - \left(\frac{250}{63} - \frac{16}{63} \right) = \frac{50}{7} - \frac{26}{7} = \frac{24}{7} \approx 3.42857143. \end{aligned}$$

Answer E. You can also do this problem using the basic definition of the expected value, but for that you must first establish the PDF of the random variable Y^P . We have

$$f_{Y^P}(y) = \frac{f_X(y+4)}{\Pr(X > 4)} = \frac{0.02 \cdot (y+4)}{1 - \int_0^4 0.02x dx} = \frac{0.02 \cdot (y+4)}{1 - (0.01x^2 \Big|_{x=0}^{x=4})} = \frac{0.02 \cdot (y+4)}{0.84} = \frac{y+4}{42}.$$

Therefor

$$E(Y^P) = \int_0^6 y \cdot \frac{y+4}{42} dy = \int_0^6 \left(\frac{y^2}{42} + \frac{2y}{21} \right) dy = \left(\frac{y^3}{126} + \frac{y^2}{21} \right) \Big|_{y=0}^{y=6} = \frac{6^3}{126} + \frac{6^2}{21} = \frac{24}{7} \approx 3.42857143.$$

Answer E, again.

© Copyright 2006 by Krzysztof Ostaszewski.

All rights reserved. Reproduction in whole or in part without express written permission from the author is strictly prohibited.

Exercises from the past actuarial examinations are copyrighted by the Society of Actuaries and/or Casualty Actuarial Society and are used here with permission.