Krzys' Ostaszewski http://www.math.ilstu.edu/krzysio/
Author of the BTDT Manual for Course P/1
available at http://smartURL.it/krzysioP or http://smartURL.it/krzysioPe
Instructor for online Course P/1 seminar: http://smartURL.it/onlineactuary
Exercise for October 8, 2005

You are given a random variable X with probability density function $f_X(x) = xe^{-x}$ for $x > 0$ and 0 otherwise. Find the first and the second moment of this probability distribution.

A. 1 and 2 B. 1 and 4 C. 2 and 2 D. 2 and 4 E. 2 and 6

Solution.
Recall the definition of the gamma function

$$\Gamma(r) = \int_0^{+\infty} x^{r-1}e^{-x}dx,$$

for and real number r other than $0,-1,-2,\ldots$ Recall also that for natural numbers n = 1, 2, …, $\Gamma(n) = (n-1)!$ Note that

$$1 = 1! = \Gamma(2) = \int_0^{+\infty} xe^{-x}dx.$$

This shows that the function given is a PDF. Furthermore,

$$E(X) = \int_0^{+\infty} x \cdot xe^{-x}dx = \int_0^{+\infty} x^2e^{-x}dx = \Gamma(3) = 2! = 2,$$

and

$$E(X^2) = \int_0^{+\infty} x^2 \cdot xe^{-x}dx = \int_0^{+\infty} x^3e^{-x}dx = \Gamma(4) = 3! = 6.$$

Answer E.