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Exercise for November 19, 2005

Casualty Actuarial Society November 2005 Course 3 Examination, Problem No. 2

Let Y_1, Y_2, Y_3, Y_4, Y_5 be the order statistics of a random sample of size 5 from a distribution having PDF $f_X(x) = e^{-x}$ for $x \geq 0$, and $f_X(x) = 0$ elsewhere. Calculate the probability that $Y_5 > 1$.

- A. Less than 0.55
- B. At least 0.55, but less than 0.65
- C. At least 0.65, but less than 0.75
- D. At least 0.75, but less than 0.85
- E. At least 0.85

Solution.

The underlying distribution is exponential with mean 1. We have

$$\begin{aligned}\Pr(Y_5 > 1) &= 1 - \Pr(Y_5 \leq 1) = \\ &= 1 - \Pr(\{Y_1 \leq 1\} \cap \{Y_2 \leq 1\} \cap \{Y_3 \leq 1\} \cap \{Y_4 \leq 1\} \cap \{Y_5 \leq 1\}) = \\ &= 1 - (F_X(1))^5 = 1 - (1 - e^{-1})^5 \approx 0.89907481.\end{aligned}$$

Answer E.

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