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Exercise for November 5, 2005
It has been determined that the probability distribution of the loss *X* for an automobile insurance policy has the density

$$f_X(x) = \frac{2}{3}e^{-x} + \frac{2}{3}e^{-2x},$$

where $x > 0$ is the amount of the loss in thousands of dollars. Find the probability that the actual loss is more than twice the expected value of the loss.

A. 0.05 B. 0.10 C. 0.14 D. 0.20 E. 0.25

Solution.
The density of the exponential distribution with mean 1 is e^{-x} for $x > 0,$ while the density of the exponential distribution with mean $\frac{1}{2}$ is $2e^{-x}$ for $x > 0.$ Since

$$f_X(x) = \frac{2}{3}e^{-x} + \frac{2}{3}e^{-2x} = \frac{2}{3}\cdot e^{-x} + \frac{1}{3}\cdot 2e^{-2x}$$

we see that the loss distribution is a mixed distribution: $\frac{2}{3}$ in the exponential distribution with mean 1 and $\frac{1}{3}$ in the exponential distribution with mean $\frac{1}{2}.$ Its mean is therefore

$$E(X) = \frac{2}{3}\cdot 1 + \frac{1}{3}\cdot\frac{1}{2} = \frac{5}{6}.$$

The probability sought is

$$\Pr\left(X > \frac{5}{3}\right) = \int_{\frac{5}{3}}^{+\infty}\left(\frac{2}{3}e^{-x} + \frac{2}{3}e^{-2x}\right)dx = \int_{\frac{5}{3}}^{+\infty}\frac{2}{3}e^{-x}dx + \int_{\frac{5}{3}}^{+\infty}\frac{2}{3}e^{-2x}dx =$$

$$= \frac{2}{3}\cdot \underbrace{\int_{\frac{5}{3}}^{+\infty} e^{-x}dx}_{\substack{\text{Probability that exponential with mean 1 is more than } \frac{5}{3}}} + \frac{1}{3}\cdot \underbrace{\int_{\frac{5}{3}}^{+\infty} 2e^{-2x}dx}_{\substack{\text{Probability that exponential with mean } \frac{1}{2} \text{ is more than } \frac{5}{3}}} = \frac{2}{3}\cdot e^{-\frac{5}{3}} + \frac{1}{3}\cdot e^{-\frac{10}{3}} \approx 0.1378084.$$

Answer C.

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