

Krzysztof Ostaszewski: <http://www.krzysio.net>

Author of the "Been There Done That!" manual for Course P/1

<http://smartURL.it/krzysioP> (paper) or <http://smartURL.it/krzysioPe> (electronic)

Instructor for Course P/1 online seminar: <http://smartURL.it/onlineactuary>

If you find these exercises valuable, please consider buying the manual or attending the seminar, and if you can't, please consider making a donation to the Actuarial Program at Illinois State University: <https://www.math.ilstu.edu/actuary/giving/>

Donations will be used for scholarships for actuarial students. Donations are tax-deductible to the extent allowed by law.

Questions about these exercises? E-mail: krzysio@krzysio.net

Exercise for December 10, 2005

Casualty Actuarial Society November 2005 Course 3 Examination, Problem No. 19

Claim size X follows a two-parameter Pareto distribution with parameters α and θ , for which the density and the cumulative distribution function are given below:

$$f_X(x) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}},$$

$$F_X(x) = 1 - \frac{\theta^\alpha}{(x+\theta)^\alpha}.$$

A transformed distribution Y is created by $Y = X^{\frac{1}{\tau}}$. Which of the following is the probability density function of Y ?

A. $f_Y(y) = \frac{\tau\theta y^{\tau-1}}{(y+\theta)^{\tau+1}}$

B. $f_Y(y) = \frac{\alpha\theta^\alpha \tau y^{\tau-1}}{(y^\tau + \theta)^{\alpha+1}}$

C. $f_Y(y) = \frac{\theta\alpha^\theta}{(y+\theta)^{\theta+1}}$

D. $f_Y(y) = \frac{\alpha\tau\left(\frac{y}{\theta}\right)^\tau}{y\left(1+\left(\frac{y}{\theta}\right)^\tau\right)^{\alpha+1}}$

E. $f_Y(y) = \frac{\alpha\theta^\alpha}{(y^\tau + \theta)^{\alpha+1}}$

Solution.

We have

$$X = Y^\tau.$$

Therefore,

$$\frac{dx}{dy} = \tau y^{\tau-1}.$$

We conclude that

$$f_Y(y) = f_X(x(y)) \cdot \left| \frac{dx}{dy} \right| = \frac{\alpha \theta^\alpha}{(y^\tau + \theta)^{\alpha+1}} \cdot \tau y^{\tau-1} = \frac{\alpha \theta^\alpha \tau y^{\tau-1}}{(y^\tau + \theta)^{\alpha+1}}.$$

Answer B.

© Copyright 2005 by Krzysztof Ostaszewski.

All rights reserved. Reproduction in whole or in part without express written permission from the author is strictly prohibited.

Exercises from the past actuarial examinations are copyrighted by the Society of Actuaries and/or Casualty Actuarial Society and are used here with permission.