

Krzysztof Ostaszewski: <http://www.math.ilstu.edu/krzysio/>
 Author of the BTDT Manual for Course P/1

available at <http://smartURL.it/krzysioP> or <http://smartURL.it/krzysioPe>
 Instructor for online Course P/1 seminar: <http://smartURL.it/onlineactuary>
 Exercise for December 17, 2005

Society of Actuaries November 2005 Course M Examination, Problem No. 8

A Mars probe has two batteries. Once a battery is activated, its future lifetime is exponential with mean 1 year. The first battery is activated when the probe lands on Mars. The second battery is activated when the first fails. Battery lifetimes after activation are independent. The probe transmits data until both batteries have failed. Calculate the probability that the probe is transmitting data three years after landing.

- A. 0.05 B. 0.10 C. 0.15 D. 0.20 E. 0.25

Solution.

Let X be the future lifetime of the first battery, and Y be the future lifetime of the second battery. We have $f_X(x) = e^{-x}$ for $x > 0$, $f_Y(y) = e^{-y}$ for $y > 0$, and also X and Y are independent. We are looking for $\Pr(X + Y > 3)$ and this can be obtained by using the convolution of these two random variables

$$f_{X+Y}(z) = \int_0^z f_X(x) \cdot f_Y(z-x) dx = \int_0^z e^{-x} \cdot e^{-(z-x)} dx = \int_0^z e^{-z} dx = ze^{-z}$$

for $z > 0$. Of course, we expected this answer, as the sum of two independent exponential random variables with the same mean is gamma with parameters $\alpha = 2$ and β equal to the mean of the exponentials. Therefore,

$$\begin{aligned} \Pr(X + Y > 3) &= \int_3^{+\infty} ze^{-z} dz = \int_0^{+\infty} (t+3)e^{-(t+3)} dt = e^{-3} \cdot \left(\int_0^{+\infty} te^{-t} dt + 3 \int_0^{+\infty} e^{-t} dt \right) = \\ &= e^{-3} \cdot \underbrace{\int_0^{+\infty} te^{-t} dt}_{\Gamma(2)=1 \text{ or mean of exponential with mean 1}} + 3e^{-3} \cdot \underbrace{\int_0^{+\infty} e^{-t} dt}_{\Gamma(1)=1 \text{ or mean of exponential with mean 1}} = 4e^{-3} \approx 0.20. \end{aligned}$$

Answer D.

© Copyright 2005 by Krzysztof Ostaszewski.

All rights reserved. Reproduction in whole or in part without express written permission from the author is strictly prohibited.

Exercises from the past actuarial examinations are copyrighted by the Society of Actuaries and/or Casualty Actuarial Society and are used here with permission.