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Exercise for December 3, 2005

November 1981 Course 110 Examination, Problem No. 37

A cookie jar has 3 red marbles and 1 white marble. A shoebox has 1 red marble and 1 white marble. Three marbles are chosen at random without replacement from the cookie jar and placed in the shoebox. Then 2 marbles are chosen at random and without replacement from the shoebox. What is the probability that both marbles chosen from the shoebox are red?

- A. $\frac{9}{10} \cdot \left(\frac{3}{4}\right)^3$ B. $\frac{43}{10}$ C. $\frac{3}{8}$ D. $\left(\frac{3}{4}\right)^3$ E. $\frac{9}{40}$

Solution.

First, when choosing three marbles from the cookie jar, there are two possible outcomes:

E_1 : all three marbles are red, leaving one white marble behind,

E_2 : two marbles are red and one marble is white, leaving one red marble behind.

But all four marbles in the jar have the same probability of being left behind, so that

$$\Pr(E_1) = \frac{1}{4},$$

$$\Pr(E_2) = \frac{3}{4}.$$

Let R be the event of choosing two red marbles from the shoebox. We have

$$\begin{aligned} \Pr(R) &= \Pr(R|E_1) \cdot \Pr(E_1) + \Pr(R|E_2) \cdot \Pr(E_2) = \\ &= \frac{\binom{4}{2}}{\binom{5}{2}} \cdot \frac{1}{4} + \frac{\binom{3}{2}}{\binom{5}{2}} \cdot \frac{3}{4} = \frac{6}{10} \cdot \frac{1}{4} + \frac{3}{10} \cdot \frac{3}{4} = \frac{15}{40} = \frac{3}{8}. \end{aligned}$$

Answer C.

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