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Exercise for February 18, 2006

May 2000 Course 3 Examination, Problem No. 25
An insurance agent will receive a bonus if his loss ratio is less than 70%. You are given:
(i) His loss ratio is calculated as incurred losses divided by earned premium on his block of business.

(ii) The agent will receive a percentage of earned premium equal to $\frac{1}{3}$ of the difference between 70% and his loss ratio.
(iii) The agent receives no bonus if his loss ratio is greater than 70%.
(iv) His earned premium is 500,000.
(v) His incurred losses are distributed according to the Pareto distribution with the cumulative distribution function:

$$F(x) = 1 - \left(\frac{600,000}{x + 600,000} \right)^3,$$

for $x > 0$. Calculate the expected value of his bonus.

A. 16,700 B. 31,500 C. 48,300 D. 50,000 E. 56,600

Solution.
Let us write X for the incurred loss and W for the value of the bonus. Note that the smallest possible value of W is 0 and the largest possible value of it is

$$\frac{1}{3} \cdot 0.7 \cdot 500,000 = \frac{350,000}{3}.$$

Then for $w > 0$ we have

$$\Pr(W > w) = \Pr\left(\frac{1}{3} \cdot \left(0.7 - \frac{X}{500,000} \right) \cdot 500,000 > w \right) = \Pr(X < 350,000 - 3w) =$$

$$= 1 - \left(\frac{600,000}{350,000 - 3w + 600,000} \right)^3 = 1 - \left(\frac{600,000}{950,000 - 3w} \right)^3.$$

We also have

$$\Pr(W = 0) = \Pr\left(\frac{X}{500000} > 0.7 \right) > 0,$$

so that the distribution of W has a point mass at zero. But that point mass has no effect on the integral of the survival function, so that

$$E(W) = \int_0^{\frac{350,000}{3}} s_W(w)\,dw = \int_0^{\frac{350,000}{3}} \left(1 - \left(\frac{600,000}{950,000 - 3w}\right)^3\right) dw =$$

$$= \frac{350,000}{3} - \left(\frac{600,000^3}{6} \cdot (950,000 - 3w)^{-2} \Big|_{w=0}^{w=\frac{350,000}{3}}\right) =$$

$$= \frac{350,000}{3} - \left(\frac{600,000^3}{6 \cdot 600,000^2} - \frac{600,000^3}{6 \cdot 950,000^2}\right) =$$

$$= \frac{350,000}{3} - 100,000 \cdot \left(1 - \left(\frac{12}{19}\right)^2\right) \approx 56,555.86.$$

Answer E.