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Exercise for February 4, 2006

You are given that for a certain insurance policy, loss X in 2005 followed the Pareto distribution with the survival function

$$s_X(x) = \left(\frac{2000}{x + 2000} \right)^2,$$

for $x > 0$. For 2006, losses increase 5% due to inflation, and an ordinary deductible of 100 is applied. Find the expected cost per loss (unconditional) in the year 2006.

- A. 2000 B. 2005 C. 2010 D. 2050 E. 2100

Solution.

Let W be the amount paid by the insurance company in 2006. We have

$$W = \begin{cases} 0, & \text{if } 1.05X \leq 100, \\ 1.05X - 100, & \text{otherwise.} \end{cases}$$

Then for any $w > 0$ we have

$$\begin{aligned} s_W(w) &= \Pr(W > w) = \Pr(1.05X - 100 > w) = \Pr\left(X > \frac{w + 100}{1.05}\right) = \\ &= \left(\frac{2000}{\frac{w + 100}{1.05} + 2000} \right)^2 = \left(\frac{2100}{w + 2200} \right)^2. \end{aligned}$$

Therefore,

$$E(W) = \int_0^{+\infty} \left(\frac{2100}{w + 2200} \right)^2 dw = \left(-\frac{2100^2}{w + 2200} \right) \Big|_{w=0}^{w \rightarrow \infty} = 0 + \frac{2100^2}{2200} \approx 2004.55.$$

Answer B.

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