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Exercise for March 11, 2006

November 1981 Course 110 Examination, Problem No. 45

Let X and Y have the joint bivariate normal distribution, where X and Y have common mean 0, common variance 1, and covariance $\frac{1}{2}$. Which of the following is equal to

$$\Pr(X + Y \leq \sqrt{3})?$$

- A. 0.11 B. 0.16 C. 0.84 D. 0.89 E. 0.96

Solution.

We have

$$E(X + Y) = E(X) + E(Y) = 0 + 0 = 0,$$

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y) = 1 + 1 + 2 \cdot \frac{1}{2} = 3,$$

and $X + Y$ has a normal distribution, because the joint distribution of X and Y is bivariate normal. Let us write Z for a standard normal random variable. Therefore,

$$\Pr(X + Y \leq \sqrt{3}) = \Pr\left(\frac{(X + Y) - 0}{\sqrt{3}} \leq \frac{\sqrt{3} - 0}{\sqrt{3}}\right) = \Pr(Z \leq 1) \approx 0.84.$$

Answer C.

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