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Exercise for March 4, 2006

November 1981 Course 110 Examination, Problem No. 42

The mean and variance of X are $\mu \neq 0$ and $\sigma^2 > 0$, respectively. If the third moment of X about the mean is $-\mu^3$, what is $E\left(X^3\right)$?

A. $\mu\sigma^2$ B. $3\sigma^2$ C. $3\mu\sigma^2$ D. $3\mu\sigma^2 - 2\mu^3$ E. $3\mu\left(\sigma^2 - \mu^2\right)$

Solution.

We have

$$E(X-\mu)^3 = E\left(X^3 - 3X^2\mu + 3X\mu^2 - \mu^3\right) = E\left(X^3\right) - 3\mu E\left(X^2\right) + 3\mu^2 E(X) - \mu^3 =$$
$$= E\left(X^3\right) - 3\mu\left(\sigma^2 + \mu^2\right) + 3\mu^2 \cdot \mu - \mu^3 = E\left(X^3\right) - 3\mu\sigma^2 - 3\mu^3 + 3\mu^3 - \mu^3 =$$
$$= E\left(X^3\right) - 3\mu\sigma^2 - \mu^3.$$

Since we know that

$$E(X-\mu)^3 = -\mu^3,$$

we conclude that

$$E\left(X^3\right) = 3\mu\sigma^2.$$

Answer C.