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Exercise for April 1, 2006

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Suppose $X_j = Z_j - Z_{j-1}$, where $j = 1, 2, \dots, n$ and $Z_0, Z_1, Z_2, \dots, Z_n$ are independent and

identically distributed with common variance σ^2 . What is $\text{Var}\left(\frac{1}{n} \sum_{j=1}^n X_j\right)$?

- A. $\frac{\sigma^2}{n}$ B. $n\sigma^2$ C. $\frac{\sigma}{\sqrt{n}}$ D. $\frac{2\sigma^2}{n^2}$ E. $\frac{2\sigma^2}{n}$

Solution.

Since any Z_j and Z_k are independent,

$$\text{Var}(Z_j - Z_k) = \sigma^2 + \sigma^2 = 2\sigma^2.$$

Therefore,

$$\begin{aligned} \text{Var}\left(\frac{1}{n} \sum_{j=1}^n X_j\right) &= \frac{1}{n^2} \text{Var}\left((Z_1 - Z_0) + (Z_2 - Z_1) + \dots + (Z_n - Z_{n-1})\right) = \\ &= \frac{1}{n^2} \text{Var}(Z_n - Z_0) = \frac{2\sigma^2}{n^2}. \end{aligned}$$

Answer D.

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