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 Exercise for April 8, 2006

X and Y have a bivariate normal distribution with $E(X) = 0$, $E(Y) = -1$, and $E(XY) = 1$.
 If $E(Y|X = 2) = 1$, and $E(X|Y = 0) = \frac{1}{16}$, find $\text{Var}(Y|X = -2)$.

- A. 3 B. 4 C. 9 D. 12 E. 15

Solution.

Let us write $E(X) = \mu_X$, $E(Y) = \mu_Y$, $\text{Var}(X) = \sigma_X^2$, $\text{Var}(Y) = \sigma_Y^2$, and

$$\rho = \rho_{YX} = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}} = \frac{E(XY) - E(X) \cdot E(Y)}{\sigma_X \cdot \sigma_Y}.$$

Note that

$$\rho = \frac{1 - 0 \cdot (-1)}{\sigma_X \cdot \sigma_Y} = \frac{1}{\sigma_X \cdot \sigma_Y}.$$

Recall that

$$E(Y|X = x) = \mu_Y + \rho \sigma_Y \cdot \frac{x - \mu_X}{\sigma_X}.$$

Therefore,

$$1 = E(Y|X = 2) = -1 + \frac{1}{\sigma_X \cdot \sigma_Y} \cdot \sigma_Y \cdot \frac{2}{\sigma_X} = -1 + \frac{2}{\sigma_X^2},$$

so that $\sigma_X = 1$. Furthermore,

$$E(X|Y = y) = \mu_X + \rho \sigma_X \cdot \frac{y - \mu_Y}{\sigma_Y}.$$

Hence,

$$\frac{1}{16} = E(X|Y = 0) = 0 + \frac{1}{\sigma_X \cdot \sigma_Y} \sigma_X \cdot \frac{0 - (-1)}{\sigma_Y} = \frac{1}{\sigma_Y^2},$$

and $\sigma_Y = 4$. This implies that $\rho = \frac{1}{\sigma_X \cdot \sigma_Y} = \frac{1}{4}$. Finally,

$$\text{Var}(Y|X = -2) = (1 - \rho^2) \sigma_Y^2 = \left(1 - \frac{1}{16}\right) \cdot 16 = 15.$$

Answer E.

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