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Author of the BTDT Manual (the “Been There Done That!” manual) for Course P/1

available at <http://smartURL.it/krzysioP> or <http://smartURL.it/krzysioPe>

Instructor for online Course P/1 seminar: <http://smartURL.it/onlineactuary>

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**Dr. Ostaszewski's online exercise posted May 7, 2005**

The time to failure  $X$  of an MP3 player follows a Weibull distribution. It is known that

$\Pr(X > 3) = \frac{1}{e}$ , and that  $\Pr(X > 6) = \frac{1}{e^4}$ . Find the probability that this MP3 player is still

functional after 4 years. Recall that the survival function of the Weibull distribution is

$s_X(x) = e^{-\left(\frac{x}{\alpha}\right)^\beta}$  for  $x > 0$ , and the distribution is nonnegative with probability one.

- A. 0.0498      B. 0.0821      C. 0.1353      D. 0.1690      E. 0.2231

Solution.

As the survival function for the Weibull distribution is  $s_X(x) = e^{-\left(\frac{x}{\alpha}\right)^\beta}$  for  $x > 0$ ,

and the problem tells us that  $s_X(3) = e^{-1}$  and  $s_X(6) = e^{-4}$ , we must have  $e^{-\left(\frac{3}{\alpha}\right)^\beta} = e^{-1}$  as

well as  $e^{-\left(\frac{6}{\alpha}\right)^\beta} = e^{-4}$ . We are looking for  $s_X(4) = e^{-\left(\frac{4}{\alpha}\right)^\beta}$ . But  $e^{-\left(\frac{3}{\alpha}\right)^\beta} = e^{-1}$  implies that

$\left(\frac{3}{\alpha}\right)^\beta = 1$ , or  $\alpha = 3$ . Furthermore,  $e^{-\left(\frac{6}{\alpha}\right)^\beta} = e^{-2^\beta} = e^{-4}$  implies that  $\beta = 2$ . Therefore, we conclude that

$$s_X(4) = e^{-\left(\frac{4}{\alpha}\right)^\beta} = e^{-\left(\frac{4}{3}\right)^2} = e^{-\frac{16}{9}} \approx 0.1690133.$$

Answer D.

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