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If you find these exercises valuable, please consider buying the manual or attending the seminar, and if you can't, please consider making a donation to the Actuarial Program at Illinois State University: https://www.math.ilstu.edu/actuary/giving/
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4. Dr. Ostaszewski's online exercise posted June 4, 2005
You are the actuary in charge of purchasing a reinsurance contract for your insurance company. You have determined that the losses that you want reinsured follow a uniform distribution on the interval [1000, 2000]. You have a choice of two reinsurance contracts for these losses. The first contract will pay 90% of the loss, while the second contract will pay up to a maximum limit, where the limit is set so that the expected payments for both contracts are the same. Find the ratio of the variance of the reinsurance payment under the second policy to the variance of the reinsurance payment under the first policy.

A. 1.5 B. 1.2 C. 0.9 D. 0.6 E. 0.3

Solution.
Let X be the loss amount. Under the first contract the expected value of the payment is 90% of the mean of X, i.e., of the uniform distribution on [1000, 2000], i.e., 90% of 1500, or 1350. The variance of the payment under the first contract is

$$\operatorname{Var}(0.9\cdot X)=0.81\cdot\operatorname{Var}(X)=0.81\cdot\frac{1000^2}{12}=67500.$$

Let us denote the policy limit under the second policy by M. The expected value of that policy's payment is:

$$\underbrace{\int_{1000}^{M}x\cdot\frac{1}{1000}dx}_{\substack{\text{For any loss amount less}\\ \text{than } M\text{, payment equals}\\ \text{to the loss, and expected}\\ \text{value calculation is done}\\ \text{under uniform distribution}}}+\underbrace{M\cdot\Pr(X\geq M)}_{\substack{\text{For any loss amount of}\\ M \text{ or more, payment is}\\ M \text{ and the probability}\\ \text{loss exceeding } M \text{ is}\\ \Pr(X\geq M)=\frac{2000-M}{1000}}}=\left(\frac{x^2}{2000}\right)\Bigg|_{x=1000}^{x=M}+M\cdot\frac{2000-M}{1000}=$$

$$=-0.0005M^2+2M-500.$$

This must be equal to 1350, i.e., $1350=-0.0005M^2+2M-500$, resulting in the equation

$$0.0005M^2-2M+1850=0,$$

so that

$$M=\frac{2\pm\sqrt{4-4\cdot 0.0005\cdot 1850}}{2\cdot 0.0005}\approx\frac{2\pm 0.5477226}{0.001}\approx\begin{cases}2547.72,\\ 1452.28.\end{cases}$$

Since the limit should not exceed 2000, $M\approx 1452.28$. We find the second moment of the payment under the second policy as:

$$\int_{1000}^{1452.28} x^2 \cdot \frac{1}{1000} dx + 1452.28^2 \cdot \Pr(X > 1452.28) =$$

$$= \left(\frac{x^3}{3000} \right) \Bigg|_{x=1000}^{x=1452.28} + 1452.28^2 \cdot 0.54772 \approx$$

$$\approx \frac{1452.28^3 - 1000^3}{3000} + 1452.28^2 \cdot 0.54772 \approx 18428812.$$

The variance of the payments of the second policy is therefore

$$18428812 - 1350^2 \approx 20382.$$

The ratio of the two variances is

$$\frac{20382}{67500} \approx 0.30.$$

Answer E.