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Exercise for July 23, 2005
Let X be a discrete random variable with moment generating function

$$M_X(t) = \frac{1}{4}\left(1 + e^{10t}\right) + \frac{1}{2} \cdot \sum_{n=0}^{+\infty} \frac{t^n}{n!}.$$

for $-\infty < t < +\infty$. Find $\Pr(X \geq 3)$.

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{3}{4}$ E. Cannot be determined

Solution.
When a random variable X is discrete and assumes values $x_1, x_2, x_3, \ldots$ with probabilities $p_1, p_2, p_3, \ldots$ then its moment generating function is

$$M_X(t) = p_1 \cdot e^{tx_1} + p_2 \cdot e^{tx_2} + p_3 \cdot e^{tx_3} + \ldots.$$

In this case, we have

$$M_X(t) = \frac{1}{4}\left(1 + e^{10t}\right) + \frac{1}{2} \cdot \sum_{n=0}^{+\infty} \frac{t^n}{n!} = \frac{1}{4} \cdot e^{0 \cdot t} + \frac{1}{4} \cdot e^{10 \cdot t} + \frac{1}{2} \cdot e^{t} = \frac{1}{4} \cdot e^{0 \cdot t} + \frac{1}{2} \cdot e^{1 \cdot t} + \frac{1}{4} \cdot e^{10 \cdot t}.$$

Therefore, the random variable whose MGF we are analyzing assumes values

$$x_1 = 0, \quad x_2 = 1, \quad \text{and} \quad x_3 = 10,$$

with probabilities

$$p_1 = \frac{1}{4}, \quad p_2 = \frac{1}{2}, \quad \text{and} \quad p_3 = \frac{1}{4},$$

respectively, so that

$$\Pr(X \geq 3) = \Pr(X = 10) = \frac{1}{4}.$$

Answer B.