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Dr. Ostaszewski's online exercise posted August 20, 2005

A random variable has the probability density function $f_X(x)=\frac{1}{2x}$, for $\frac{1}{e}\le x\le e$, and 0 otherwise. What is the probability that among of four independent observations of *X* there are three less than 1 and one that is greater than 1?

A. $\frac{1}{16}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{3}{4}$ E. $\frac{7}{8}$

Solution.
We have

$$\Pr(X<1)=\int_{\frac{1}{e}}^{1}\frac{1}{2x}dx=\int_{\frac{1}{e}}^{1}\frac{1}{2}\cdot\frac{1}{x}dx=\frac{1}{2}\cdot\ln x\Big|_{x=\frac{1}{e}}^{x=1}=\frac{1}{2}\cdot\left(\ln 1-\ln\left(\frac{1}{e}\right)\right)=\frac{1}{2}\cdot\ln\left(-(-1)\right)=\frac{1}{2}.$$

Each observation of this random variable being less than 1 has the probability of $\frac{1}{2}$. Four independent observations of this random variable, considered only as to whether each is less than 1 or not, can be treated as four Bernoulli Trials with probability of success of $\frac{1}{2}$ and the probability of failure of $\frac{1}{2}$. The probability of exactly three successes in four such Bernoulli Trials is given by the binomial formula

$$\binom{4}{3}\cdot\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)^1=\frac{4}{16}=\frac{1}{4}.$$

Answer B.