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**Dr. Ostaszewski's online exercise posted August 27, 2005**

A machine consists of two components. The lifetimes of the two components are identically distributed with the following common density function

$$f_X(x) = \frac{3}{4}e^{-3x} + \frac{3}{2}e^{-2x}$$

for $x > 0$, and 0 otherwise. The machine breaks down if any of the two components fails. Find the expected lifetime of the device.

A. 0.1124 B. 0.2260 C. 0.2981 D. 0.3267 E. 0.3999

Solution.

The density can be written as $f_X(x) = \frac{3}{4}e^{-3x} + \frac{3}{2}e^{-2x} = \frac{1}{4}\cdot 3e^{-3x} + \frac{3}{4}\cdot 2e^{-2x}$, and therefore this is a mixed distribution, with a weight of 0.25 in an exponential with hazard rate 3 and a weight of 0.75 in an exponential with hazard rate 2. The survival function of this distribution is the same kind of mixture of the survival functions of the two exponential distributions. Let $X_1$ be the lifetime of the first component, $X_2$ be the lifetime of the second component, and $Y$ be the lifetime of the device. Then $Y = \min(X_1, X_2)$. Therefore

$$s_Y(y) = \Pr(Y > y) = \Pr(\min(X_1, X_2) > y) = \Pr(\{X_1 > y\} \cap \{X_2 > y\}) =$$

$$= \Pr(X_1 > y)\cdot \Pr(X_2 > y) = \left(\frac{1}{4}\cdot e^{-3y} + \frac{3}{4}\cdot e^{-2y}\right)^2.$$

The expected value of $Y$ is

$$E(Y) = \int_0^{+\infty}\left(\frac{1}{4}\cdot e^{-3y} + \frac{3}{4}\cdot e^{-2y}\right)^2 dy = \int_0^{+\infty}\left(\frac{1}{16}\cdot e^{-6y} + \frac{9}{16}\cdot e^{-4y} + \frac{6}{16}\cdot e^{-5y}\right)dy =$$

$$= \frac{1}{16}\cdot\frac{1}{6} + \frac{9}{16}\cdot\frac{1}{4} + \frac{6}{16}\cdot\frac{1}{5} = \frac{1}{16}\cdot\left(\frac{10}{60} + \frac{135}{60} + \frac{72}{60}\right) = \frac{1}{16}\cdot\frac{217}{60} = \frac{217}{960} \approx 0.2260.$$

Answer B.