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Exercise for August 6, 2005
Let N_1 and N_2 be two independent Poisson random variables with expected values $E(N_1)=2$ and $E(N_2)=3$. Find $\operatorname{Var}(N_1 | N_1+N_2=5)$.

A. 0.4 B. 1.0 C. 1.2 D. 2.4 E. 4.8

Solution.
Let $\lambda_1 = 2$ and $\lambda_2 = 3$. Because both of these random variables are non-negative, if $N_1+N_2=5$, N_1 can only assume values of 0, 1, 2, 3, 4, or 5. Because N_1 and N_2 are independent and Poisson, N_1+N_2 is Poisson with mean 2 + 3 = 5, so that

$$\Pr(N_1+N_2=5)=\frac{5^5}{5!}\cdot e^{-5}.$$

Therefore, for k = 0, 1, 2, 3, 4, or 5, we have

$$\Pr(N_1=k|N_1+N_2=5)=\frac{\Pr(\{N_1=k\}\cap\{N_1+N_2=5\})}{\Pr(N_1+N_2=5)}=$$

$$=\frac{\Pr(N_1=k)\cdot\Pr(N_2=5-k)}{\Pr(N_1+N_2=5)}=$$

$$=\frac{\frac{2^k}{k!}\cdot e^{-2}\cdot\frac{3^{5-k}}{(5-k)!}\cdot e^{-3}}{\frac{5^5}{5!}\cdot e^{-5}}=\binom{5}{k}\cdot\left(\frac{2}{5}\right)^k\cdot\left(\frac{3}{5}\right)^{5-k}.$$

Therefore, $N_1|N_1+N_2=5$ is binomial with $p=\frac{2}{5}$ and $n=5$ and its variance is

$$5\cdot\frac{2}{5}\cdot\frac{3}{5}=\frac{6}{5}=1.2.$$

Answer C.