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Exercise for September 3, 2005

Let X be an exponential random variable with mean 2. Define a new random variable $Y = \min(X^2, 2)$. Find $F_Y(1)$.

- A. 0 B. 1 C. $\frac{1}{e}$ D. $\frac{1}{\sqrt{e}}$ E. $1 - \frac{1}{\sqrt{e}}$

Solution.

We have

$$Y = \min(X^2, 2) = \begin{cases} X^2, & \text{for } 0 < X \leq \sqrt{2}, \\ 2, & \text{for } X > \sqrt{2}. \end{cases}$$

By definition of the cumulative distribution function

$$F_Y(1) = \Pr(Y \leq 1) = \Pr(\{X^2 \leq 1\} \cap \{0 < X \leq \sqrt{2}\}) = \Pr(X \leq 1) = 1 - e^{-\frac{1}{2}}.$$

Answer E.

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