November 2005 Course FM/2 Examination

1. An insurance company earned a simple rate of interest of 8% over the last calendar year based on the following information:
   Assets, beginning of year 25,000,000
   Sales revenue X
   Net investment income 2,000,000
   Salaries paid 2,200,000
   Other expenses paid 750,000
All cash flows occur at the middle of the year. Calculate the effective yield rate.
A. 7.7%  B. 7.8%  C. 7.9%  D. 8.0%  E. 8.1%

2. Calculate the Macaulay duration of an eight-year 100 par value bond with 10% annual coupons and an effective rate of interest equal to 8%.
A. 4  B. 5  C. 6  D. 7  E. 8

3. An investor accumulates a fund by making payments at the beginning of each month for 6 years. Her monthly payment is 50 for the first 2 years, 100 for the next 2 years, and 150 for the last 2 years. At the end of the 7th year the fund is worth 10,000. The annual effective interest rate is \( i \), and the monthly effective interest rate is \( j \). Which of the following formulas represents the equation of value for this fund accumulation?
A. \( s_{7i} \cdot (1 + i) \cdot \left( (1 + i)^4 + 2(1 + i)^2 + 3 \right) = 200 \)
B. \( s_{7j} \cdot (1 + j) \cdot \left( (1 + j)^4 + 2(1 + j)^2 + 3 \right) = 200 \)
C. \( s_{7i} \cdot (1 + i) \cdot \left( (1 + i)^4 + 2(1 + i)^2 + 3 \right) = 200 \)
D. \( s_{7j} \cdot (1 + j) \cdot \left( (1 + j)^4 + 2(1 + j)^2 + 3 \right) = 200 \)
E. \( s_{7j} \cdot (1 + j) \cdot \left( (1 + j)^4 + 2(1 + j)^2 + 3 \right) = 200 \)

4. A ten-year 100 par value bond pays 8% coupons semiannually. The bond is priced at 118.20 to yield an annual nominal rate of 6% convertible semiannually. Calculate the redemption value of the bond.
A. 97  B. 100  C. 103  D. 106  E. 109
5. Alex is an investment analyst for a large fund management firm. He specializes in finding risk-free arbitrage opportunities in the stock market. His strategy consists of selling a specific number of call options for each share of stock selected in the fund. Which of the following best describes the technique used by Alex to achieve his goal?

A. Black-Scholes option pricing model  
B. Capital Asset Pricing Model  
C. Full immunization  
D. Short sales  
E. Hedge ratio

6. Consider a yield curve defined by the following equation:

\[ i_k = 0.09 + 0.002k - 0.001k^2, \]

where \( i_k \) is the annual effective rate of return for zero coupon bonds with maturity of \( k \) years. Let \( j \) be the one-year effective rate during year 5 that is implied by this yield curve. Calculate \( j \).

A. 4.7%  
B. 5.8%  
C. 6.6%  
D. 7.5%  
E. 8.2%

7. A bank offers the following choices for certificates of deposit:

<table>
<thead>
<tr>
<th>Term (in years)</th>
<th>Nominal annual interest rate convertible quarterly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00%</td>
</tr>
<tr>
<td>3</td>
<td>5.00%</td>
</tr>
<tr>
<td>5</td>
<td>5.65%</td>
</tr>
</tbody>
</table>

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates. An investor initially deposits 10,000 in the bank and withdraws both principal and interest at the end of 6 years. Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.

A. 5.09%  
B. 5.22%  
C. 5.35%  
D. 5.48%  
E. 5.61%

8. Matthew makes a series of payments at the beginning of each year for 20 years. The first payment is 100. Each subsequent payment through the tenth year increases by 5% from the previous payment. After the tenth payment, each payment decreases by 5% from the previous payment. Calculate the present value of these payments at the time the first payment is made using an annual effective rate of 7%.

A. 1375  
B. 1385  
C. 139  
D. 1405  
E. 1415

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9. A company deposits 1000 at the beginning of the first year and 150 at the beginning of each subsequent year into perpetuity. In return the company receives payments at the end of each year forever. The first payment is 100. Each subsequent payment increases by 5%. Calculate the company’s yield rate for this transaction.

A. 4.7%  B. 5.7%  C. 6.7%  D. 7.7%  E. 8.7%

10. A company must pay liabilities of 1000 and 2000 at the end of years 1 and 2, respectively. The only investments available to the company are the following two zero-coupon bonds:

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Effective annual yield</th>
<th>Par</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>12%</td>
<td>1000</td>
</tr>
</tbody>
</table>

Determine the cost to the company to match its liabilities exactly.

A. 2007  B. 2259  C. 2503  D. 2756  E. 3001

11. An investor borrows an amount at an annual effective interest rate of 5% and will repay all interest and principal in a lump sum at the end of 10 years. She uses the amount borrowed to purchase a 1000 par value 10-year bond with 8% semiannual coupons bought to yield 6% convertible semiannually. All coupon payments are reinvested at a nominal rate of 4% convertible semiannually. Calculate the net gain to the investor at the end of 10 years after the loan is repaid.

A. 96  B. 101  C. 106  D. 111  E. 116

12. Megan purchases a perpetuity-immediate for 3250 with annual payments of 130. At the same price and interest rate, Chris purchases an annuity-immediate with 20 annual payments that begin at amount $P$ and increase by 15 each year thereafter. Calculate $P$.

A. 90  B. 116  C. 131  D. 176  E. 239

13. For 10,000, Kelly purchases an annuity-immediate that pays 400 quarterly for the next 10 years. Calculate the annual nominal interest rate convertible monthly earned by Kelly’s investment.

A. 10.0%  B. 10.3%  C. 10.5%  D. 10.7%  E. 11.0%

14. Payments of $X$ are made at the beginning of each year for 20 years. These payments earn interest at the end of each year at an annual effective rate of 8%. The interest is immediately reinvested at an annual effective rate of 6%. At the end of 20 years, the...
accumulated value of the 20 payments and the reinvested interest is 5600. Calculate \( X \).

A. 121.67     B. 123.56     C. 125.72     D. 127.18     E. 128.50

15. You are given the following term structure of spot interest rates:

<table>
<thead>
<tr>
<th>Term (in years)</th>
<th>Spot interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00%</td>
</tr>
<tr>
<td>2</td>
<td>5.75%</td>
</tr>
<tr>
<td>3</td>
<td>6.25%</td>
</tr>
<tr>
<td>4</td>
<td>6.50%</td>
</tr>
</tbody>
</table>

A three-year annuity-immediate will be issued a year from now with annual payments of 5000. Using the forward rates, calculate the present value of this annuity a year from now.

A. 13,094     B. 13,153     C. 13,296     D. 13,321     E. 13,401

16. Dan purchases a 1000 par value 10-year bond with 9% semiannual coupons for 925. He is able to reinvest his coupon payments at a nominal rate of 7% convertible semiannually. Calculate his nominal annual yield rate convertible semiannually over the ten-year period.

A. 7.6%     B. 8.1%     C. 9.2%     D. 9.4%     E. 10.2%

17. Theo sells a stock short with a current price of 25,000 and buys it back for \( X \) at the end of 1 year. Governmental regulations require the short seller to deposit margin of 40% at the time of the short sale. The prevailing interest rate is an 8% annual rate, and Theo earns a 25% yield on the transaction. Calculate \( X \).

A. 19,550     B. 20,750     C. 22,500     D. 23,300     E. 24,500

18. A loan is repaid with level annual payments based on an annual effective interest rate of 7%. The 8th payment consists of 789 of interest and 211 of principal. Calculate the amount of interest paid in the 18th payment.

A. 415     B. 444     C. 556     D. 585     E. 612

19. Which of the following statements about zero-coupon bonds are true?

I. Zero-coupon bonds may be created by separating the coupon payments and redemption values from bonds and selling each of them separately.

II. The yield rates on stripped Treasuries at any point in time provide an immediate reading of the risk-free yield curve.
III. The interest rates on the risk-free yield curve are called forward rates.

A. I only
B. II only
C. III only
D. I, II, and III
E. The correct answer is not given by A, B, C, or D.

20. The dividends of a common stock are expected to be 1 at the end of each of the next 5 years and 2 for each of the following 5 years. The dividends are expected to grow at a fixed rate of 2% per year thereafter. Assume an annual effective interest rate of 6%. Calculate the price of this stock using the dividend discount model.

A. 29 B. 33 C. 37 D. 39 E. 41

21. Which of the following statements about immunization strategies are true?
I. To achieve immunization, the convexity of the assets must equal the convexity of the liabilities.
II. The full immunization technique is designed to work for any change in the interest rate.
III. The theory of immunization was developed to protect against adverse effects created by changes in interest rates.

A. None B. I and II only C. I and III only D. II and III only E. The correct answer is not given by A, B, C, and D.

22. A 1000 par value bond with coupons at 9% payable semiannually was called for 1100 prior to maturity. The bond was bought for 918 immediately after a coupon payment and was held to call. The nominal yield rate convertible semiannually was 10%. Calculate the number of years the bond was held.

A. 10 B. 25 C. 39 D. 49 E. 54

23. The present value of a 25-year annuity-immediate with a first payment of 2500 and decreasing by 100 each year thereafter is X. Assuming an annual effective interest rate of 10%, calculate X.

A. 11,346 B. 13,615 C. 15,923 D. 17,396 E. 18,112

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24. A 30-year bond with a par value of 1000 and 12% coupons payable quarterly is selling at 850. Calculate the annual nominal yield rate convertible quarterly.
A. 3.5%  B. 7.1%  C. 14.2%  D. 14.9%  E. 15.4%

25. The parents of three children, ages 1, 3, and 6, wish to set up a trust fund that will pay $X$ to each child upon attainment of age 18, and $Y$ to each child upon attainment of age 21. They will establish the trust fund with a single investment of $Z$. Which of the following is the correct equation of value for $Z$?

A. \( \frac{X}{v^{17} + v^{15} + v^{12}} + \frac{Y}{v^{20} + v^{18} + v^{15}} \)
B. \( 3 \cdot \left( Xv^{18} + Yv^{21} \right) \)
C. \( 3 \cdot Xv^3 + Y \cdot \left( v^{20} + v^{18} + v^{15} \right) \)
D. \( \left( X + Y \right) \cdot \frac{v^{20} + v^{18} + v^{15}}{v^3} \)
E. \( X \cdot \left( v^{17} + v^{15} + v^{12} \right) + Y \cdot \left( v^{20} + v^{18} + v^{15} \right) \)
1. November 2005 Course FM/2 Examination, Problem No. 1, and Dr. Ostaszewski’s online exercise No. 233 posted August 22, 2009
An insurance company earned a simple rate of interest of 8% over the last calendar year based on the following information:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets, beginning of year</td>
<td>25,000,000</td>
</tr>
<tr>
<td>Sales revenue</td>
<td>X</td>
</tr>
<tr>
<td>Net investment income</td>
<td>2,000,000</td>
</tr>
<tr>
<td>Salaries paid</td>
<td>2,200,000</td>
</tr>
<tr>
<td>Other expenses paid</td>
<td>750,000</td>
</tr>
</tbody>
</table>

All cash flows occur at the middle of the year. Calculate the effective yield rate.

A. 7.7%  B. 7.8%  C. 7.9%  D. 8.0%  E. 8.1%

Solution.
You are supposed to guess that net investment income is not a cash flow, while only these accounting entries: sales revenue, salaries paid, and other expenses paid, contribute to the cash flows. On the other hand, you are supposed to include net investment income as income obtained at the end of the year (not before, as it apparently did not result in a cash flow). The net cash flow occurring in the middle of the year is

\[
X - 2,200,000 - 750,000 = X - 2,950,000.
\]

Therefore, the rate of return based on simple interest, i.e., the dollar-weighted rate of return, is

\[
\frac{\text{Middle-year cash flow}}{X - 2,950,000} + \frac{\text{Year-end investment income}}{2,000,000} = 0.08.
\]

As 8% of 25 million is 2 million, this results in

\[
2,000,000 + 0.04 \cdot (X - 2,950,000) = X - 950,000,
\]

or

\[
0.04 \cdot (X - 2,950,000) = 1 \cdot (X - 2,950,000).
\]

But we know that 0.04 is not equal to one, so unless \(X = 2,950,000\), the above equation results in a contradiction. Hence \(X = 2,950,000\) and the middle-year cash flow is zero. This means that the initial 25,000,000 brings 2,000,000 of income during the year, resulting in the effective yield rate of 8%.

Answer D.
Calculate the Macaulay duration of an eight-year 100 par value bond with 10% annual coupons and an effective rate of interest equal to 8%.

A. 4  B. 5  C. 6  D. 7  E. 8

Solution.
The Macaulay duration of this bond equals, by definition,

\[
\frac{1 \cdot 10 \cdot 1.08^{-1} + 2 \cdot 10 \cdot 1.08^{-2} + \ldots + 8 \cdot 10 \cdot 1.08^{-8} + 8 \cdot 100 \cdot 1.08^{-8}}{10 \cdot 1.08^{-1} + 10 \cdot 1.08^{-2} + \ldots + 10 \cdot 1.08^{-8} + 100 \cdot 1.08^{-8}} =
\]

\[
\frac{10 \cdot (la)_{8|0.08} + 800 \cdot 1.08^{-8}}{10 \cdot a_{8|0.08} + 100 \cdot 1.08^{-8}} = \frac{10 \cdot 0.08 + 800 \cdot 1.08^{-8}}{10 \cdot 1.08 + 100 \cdot 1.08^{-8}} \approx 5.9891.
\]

You can also calculate for the portion of this bond that pays an 8% coupon, which represents a bond trading at par, as having the price of 100, and Macaulay duration of \( \ddot{a}_{8|0.08} \approx 6.2064 \), while the additional payment of 2 per year over eight years has the value of \( 2a_{8|0.08} = 2 \ddot{a}_{8|0.08} \cdot 1.08^{-1} \approx 11.4933 \), and the Macaulay duration of (note that we have \( m = 1, n = 8, d = 0.08 \cdot 1.08^{-1} \))

\[
\frac{1}{d^{(m)}} = \frac{n}{(1+i)^n - 1} = \frac{1.08}{0.08} \frac{8}{1.08 - 1} \approx 4.0985.
\]

Thus the Macaulay duration of the bond can be calculated as the weighted average of its two pieces

\[
\frac{100 \cdot 6.2064 + 11.4933 \cdot 4.0985}{111.4933} \approx 5.9891.
\]

Answer C.

3. November 2005 Course FM/2 Examination, Problem No. 3, and Dr. Ostaszewski’s online exercise No. 258 posted April 24, 2010

An investor accumulates a fund by making payments at the beginning of each month for 6 years. Her monthly payment is 50 for the first 2 years, 100 for the next 2 years, and 150 for the last 2 years. At the end of the 7th year the fund is worth 10,000. The annual effective interest rate is \( i \), and the monthly effective interest rate is \( j \). Which of the following formulas represents the equation of value for this fund accumulation?

A. \( \ddot{s}_{2|0.08} \cdot (1+i) \cdot \left((1+i)^4 + 2(1+i)^2 + 3\right) = 200 \)
B. \( \ddot{s}_{2|0.08} \cdot (1+j) \cdot \left((1+j)^4 + 2(1+j)^2 + 3\right) = 200 \)
C. \( \ddot{s}_{2|0.08} \cdot (1+i) \cdot \left((1+i)^4 + 2(1+i)^2 + 3\right) = 200 \)
D. \( s_{24|4} \cdot (1 + i) \cdot \left( (1 + i)^4 + 2(1 + i)^2 + 3 \right) = 200 \)

E. \( s_{24|4} \cdot (1 + j) \cdot \left( (1 + j)^4 + 2(1 + j)^2 + 3 \right) = 200 \)

Solution.
After 24 months, the account will accumulate to \( 50s_{24|y} \). After additional 24 months, the accumulation will be \( 50s_{24|y} \cdot (1 + i)^2 + 100s_{24|y} \). After additional 24 months, the accumulated value will be
\[ 50s_{24|y} \cdot (1 + i)^4 + 100s_{24|y} \cdot (1 + i)^2 + 150s_{24|y} = 50s_{24|y} \cdot \left( (1 + i)^4 + 2(1 + i)^2 + 3 \right). \]
After one more year, at the end of the seventh year, the value will be
\[ 50s_{24|y} \cdot (1 + i) \cdot \left( (1 + i)^4 + 2(1 + i)^2 + 3 \right), \]
and we know this to be 10,000. Therefore, the equation of value is
\[ s_{24|y} \cdot (1 + i) \cdot \left( (1 + i)^4 + 2(1 + i)^2 + 3 \right) = \frac{10,000}{50} = 200. \]
Answer C.

4. November 2005 Course FM/2 Examination, Problem No. 4, and Dr. Ostaszewski’s online exercise No. 259 posted May 1, 2010
A ten-year 100 par value bond pays 8% coupons semiannually. The bond is priced at 118.20 to yield an annual nominal rate of 6% convertible semiannually. Calculate the redemption value of the bond.

A. 97  B. 100  C. 103  D. 106  E. 109

Solution.
Based on the Frank formula, \( P = Fr \cdot a_{\frac{n}{2}} + K \), we obtain the equation
\[ 118.20 = 4 \cdot a_{\frac{20}{3\%}} + C \cdot 1.03^{-20}. \]
This can be solved using a financial calculator with \( n = 20 \), \( i = 3\% \), \( PV = 118.20 \), \( PMT = 4 \), resulting in \( C \approx 106.00 \). Alternatively, solving the equation
\[ C = 118.20 \cdot 1.03^{-20} - 4 \cdot s_{\frac{20}{3\%}} \approx 106.00. \]
Answer D.

5. November 2005 Course FM/2 Examination, Problem No. 5, and Dr. Ostaszewski’s online exercise No. 260 posted May 8, 2010
Alex is an investment analyst for a large fund management firm. He specializes in finding risk-free arbitrage opportunities in the stock market. His strategy consists of selling a specific number of call options for each share of stock selected in the fund. Which of the following best describes the technique used by Alex to achieve his goal?
Solution.
This question was not counted as it referred to concepts not covered in the syllabus. But let us go over the answers and treat this as a learning experience:
A. Black-Scholes option pricing model: This is not in the syllabus of Course FM/2. My guess is that the “original intent” of the question creator was that you would see this and realize it is not in the syllabus, and reject this answer. Same as if the answer were: The Easter Bunny. In any case, the Black-Scholes option pricing model could mean modeling options stochastically (general concept of a Black-Scholes option pricing model), or the Black-Scholes option pricing formula. The formula can actually be used to derive a hedge ratio in hedging stock position with options and vice versa. So this answer does describe the technique used by Alex to achieve his goal, it just does not do it the best way. And it is an Easter Bunny answer, i.e., it was not covered in the syllabus.
B. Capital Asset Pricing Model. This is yet another Easter Bunny answer. CAPM is not in the syllabus. CAPM is the theory that gives the expected rate of return on a stock in a one-period model of a market in relation to the risk-free rate of return and the rate of return on the entire market portfolio. Not here.
C. Full immunization is in the Course FM/2 syllabus. It refers to the technique of interest rate risk management in which asset portfolio duration is set equal to the liabilities portfolio duration, while asset portfolio convexity exceeds the convexity of the liabilities portfolio, and assets value equals or exceeds the value of liabilities. Not here.
D. Short sales. As short sales are presented in the Course FM/2 syllabus, you would not consider anything in this question to involve short sales. However, Alex has a short position in options, as he is selling them. But these are not really “short sales.” So this is not the right answer, either.
E. Hedge ratio. Buying a security, whose values move in opposite direction of that of the currently held portfolio, is called hedging. It is a risk-management strategy. The ratio of the number of units of the security bought to the number of units of the security currently held is called the hedge ratio. This is the right answer here, as the short calls position will move in opposite direction of the movements of the underlying stock, and in order to specify the number of option contracts to be written, Alex must calculate the hedge ratio. So this answer makes the most sense.
Answer E.

6. November 2005 Course FM/2 Examination, Problem No. 6, and Dr. Ostaszewski’s online exercise No. 261 posted May 15, 2010
Consider a yield curve defined by the following equation \( i_k = 0.09 + 0.002k - 0.001k^2 \), where \( i_k \) is the annual effective rate of return for zero coupon bonds with maturity of \( k \) years. Let \( j \) be the one-year effective rate during year 5 that is implied by this yield curve.
Calculate \( j \).

A. 4.7%  
B. 5.8%  
C. 6.6%  
D. 7.5%  
E. 8.2%

Solution.
The question effectively asks you to find the forward rate \( f_5 \) from time 4 to time 5. That forward rate can be calculated from the spot rates as \( f_5 = \frac{(1+i_5)^5}{(1+i_4)^4} - 1 \). This equals

\[
\frac{(1+0.09+0.01-0.025)^5}{(1+0.09+0.008-0.016)^4} - 1 = 1.075^5 - 1 = 4.744994\%.
\]

Answer A.

7. November 2005 Course FM/2 Examination, Problem No. 7, and Dr. Ostaszewski’s online exercise No. 262 posted May 22, 2010

A bank offers the following choices for certificates of deposit:

<table>
<thead>
<tr>
<th>Term (in years)</th>
<th>Nominal annual interest rate convertible quarterly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00%</td>
</tr>
<tr>
<td>3</td>
<td>5.00%</td>
</tr>
<tr>
<td>5</td>
<td>5.65%</td>
</tr>
</tbody>
</table>

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates. An investor initially deposits 10,000 in the bank and withdraws both principal and interest at the end of 6 years. Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.

A. 5.09%  
B. 5.22%  
C. 5.35%  
D. 5.48%  
E. 5.61%

Solution.
The investor has the following options:

• Deposit the funds in one-year certificates every year for six years, producing an annual effective rate of return of

\[
\left(1 + \frac{0.04}{4}\right)^{4\times6} - 1 = 1.01^{24} - 1 \approx 4.060401\%.
\]

• Deposit the funds for three years at 5% per year convertible quarterly, followed by three one-year deposits at 4% convertible quarterly, resulting in

\[
\left\{(1 + \frac{0.05}{4})^{4\times3}\right\} \times \left\{\left(1 + \frac{0.04}{4}\right)^{1}\right\}^{\frac{1}{6}} - 1 \approx 4.576189\%.
\]

• Deposit the funds for three years at 5% per year convertible quarterly, followed by the same deposit for another three years, resulting in an annual effective rate of return of
\[
\left( \left( 1 + \frac{0.05}{4} \right)^{4} \right)^{\frac{1}{6}} - 1 = 1.014^\frac{1}{6} - 1 \approx 5.094534\%.
\]

• Deposit the funds for five years, and then for one year, or deposit funds for one year and then for five years, producing the same annual effective rate of return of

\[
\left( \left( 1 + \frac{0.0565}{4} \right)^{4} \right)^{\frac{1}{6}} - 1 \approx 5.483827\%.
\]

The last one is the best choice. Given the choices, this was to be expected: you could have guessed intuitively that this is the highest effective rate you can get. The most effective way to help develop that guess is to calculate the effective annual rate of return on each of the available certificates:

• One-year certificate: \( 1 + \frac{0.04}{4} \) \( - 1 \approx 4.060401\% \).

• Three-year certificate: \( 1 + \frac{0.05\cdot 4}{4} \) \( - 1 \approx 5.094534\% \).

• Five-year certificate: \( 1 + \frac{0.0565\cdot 4}{4} \) \( - 1 \approx 5.770841\% \).

Clearly, getting roughly 5.77% per year for five years makes up for getting 4.06% for just one year versus the second best of getting 5.09% per year.

Answer D.

8. November 2005 Course FM/2 Examination, Problem No. 8, and Dr. Ostaszewski's online exercise No. 263 posted May 29, 2010

Matthew makes a series of payments at the beginning of each year for 20 years. The first payment is 100. Each subsequent payment through the tenth year increases by 5% from the previous payment. After the tenth payment, each payment decreases by 5% from the previous payment. Calculate the present value of these payments at the time the first payment is made using an annual effective rate of 7%.

A. 1375 B. 1385 C. 1395 D. 1405 E. 1415

Solution.

The present value of the first ten payments is

\[
100 + 100 \cdot \frac{1.05}{1.07} + \ldots + 100 \cdot \frac{1.05^9}{1.07^9} = 100 \cdot \frac{1 - \left( \frac{1.05}{1.07} \right)^{10}}{1 - \frac{1.05}{1.07}}.
\]

The present value of the final ten payments is
A company deposits 1000 at the beginning of the first year and 150 at the beginning of each subsequent year into perpetuity. In return the company receives payments at the end of each year forever. The first payment is 100. Each subsequent payment increases by 5%. Calculate the company’s yield rate for this transaction.

A. 4.7%  
B. 5.7%  
C. 6.7%  
D. 7.7%  
E. 8.7%

Solution.
Let \( j \) be the annual yield rate sought. We have

\[
1000 + \frac{150}{j} = \frac{100}{1 + j} + \frac{100 \cdot 1.05}{(1 + j)^2} + \frac{100 \cdot 1.05^2}{(1 + j)^3} + \ldots = \\
= \frac{100}{1 + j} \sum_{n=0}^{\infty} \left( \frac{1.05}{1 + j} \right)^n = \frac{100}{1 + j} \cdot \frac{1}{1 - \frac{1.05}{1 + j}} = \frac{100}{j - 0.05}.
\]

Therefore,

\[
\frac{1000 j + 150}{j} = \frac{100}{j - 0.05},
\]
or

\[
1000 j^2 + 150 j - 50 j - 7.5 = 100 j,
\]
or

\[
100 j^2 - 7.5 = 0, \text{ so that}
\]

\[
j = \sqrt{\frac{7.5}{1000}} = \sqrt{0.0075} = 8.660254%.
\]

Answer E.

9. November 2005 Course FM/2 Examination, Problem No. 9, and Dr. Ostaszewski’s online exercise No. 264 posted June 5, 2010

A company deposits 1000 at the beginning of the first year and 150 at the beginning of each subsequent year into perpetuity. In return the company receives payments at the end of each year forever. The first payment is 100. Each subsequent payment increases by 5%. Calculate the company’s yield rate for this transaction.

A. 4.7%  
B. 5.7%  
C. 6.7%  
D. 7.7%  
E. 8.7%

Solution.
Let \( j \) be the annual yield rate sought. We have

\[
1000 + \frac{150}{j} = \frac{100}{1 + j} + \frac{100 \cdot 1.05}{(1 + j)^2} + \frac{100 \cdot 1.05^2}{(1 + j)^3} + \ldots = \\
= \frac{100}{1 + j} \sum_{n=0}^{\infty} \left( \frac{1.05}{1 + j} \right)^n = \frac{100}{1 + j} \cdot \frac{1}{1 - \frac{1.05}{1 + j}} = \frac{100}{j - 0.05}.
\]

Therefore,

\[
\frac{1000 j + 150}{j} = \frac{100}{j - 0.05},
\]
or

\[
1000 j^2 + 150 j - 50 j - 7.5 = 100 j,
\]
or

\[
100 j^2 - 7.5 = 0, \text{ so that}
\]

\[
j = \sqrt{\frac{7.5}{1000}} = \sqrt{0.0075} = 8.660254%.
\]

Answer E.
10. November 2005 Course FM/2 Examination, Problem No. 10, and Dr. Ostaszewski’s online exercise No. 265 posted June 12, 2010
A company must pay liabilities of 1000 and 2000 at the end of years 1 and 2, respectively. The only investments available to the company are the following two zero-coupon bonds:

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Effective annual yield</th>
<th>Par</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>12%</td>
<td>1000</td>
</tr>
</tbody>
</table>

Determine the cost to the company to match its liabilities exactly.

A. 2007 B. 2259 C. 2503 D. 2756 E. 3001

Solution.
As assets match the liabilities, they will have exactly the same cash flows and the same present value. Therefore, the cost is

\[
\frac{1000}{1.10} + \frac{2000}{1.12^2} = 2503.49.
\]

Answer C.

11. November 2005 Course FM/2 Examination, Problem No. 11, and Dr. Ostaszewski’s online exercise No. 266 posted June 19, 2010
An investor borrows an amount at an annual effective interest rate of 5% and will repay all interest and principal in a lump sum at the end of 10 years. She uses the amount borrowed to purchase a 1000 par value 10-year bond with 8% semiannual coupons bought to yield 6% convertible semiannually. All coupon payments are reinvested at a nominal rate of 4% convertible semiannually. Calculate the net gain to the investor at the end of 10 years after the loan is repaid.

A. 96 B. 101 C. 106 D. 111 E. 116

Solution.
The price of the bond purchased, which equals the principal amount borrowed, is

\[
40a_{20|3\%} + 1000 \cdot 1.03^{-20} \approx 1148.77.
\]

This is also amount borrowed. Therefore, the amount repaid at the end of 10 years is

\[
1148.77 \cdot 1.05^{10} \approx 1871.23.
\]

The coupons reinvested accumulate to

\[
40s_{20|2\%} \approx 971.89.
\]

At the end of ten years, the investor will receive 1000 maturity payment and 971.89 from the accumulation of coupons, but will have to repay 1871.23, for a net of

\[
1000 + 971.89 - 1871.23 = 100.66.
\]

Answer B.
12. November 2005 Course FM/2 Examination, Problem No. 12, and Dr. Ostaszewski’s online exercise No. 267 posted June 26, 2010
Megan purchases a perpetuity-immediate for 3250 with annual payments of 130. At the same price and interest rate, Chris purchases an annuity-immediate with 20 annual payments that begin at amount $P$ and increase by 15 each year thereafter. Calculate $P$.

A. 90  	B. 116  	C. 131  	D. 176  	E. 239

Solution.
If $i$ is the interest rate used for both the perpetuity purchased by Megan and the annuity purchased by Chris, based on the information given for the perpetuity we have
\[
\frac{130}{i} = 3250, \\
\text{so that } i = \frac{130}{3250} = 0.04.
\]
Chris’ annuity consists of a level 20-year annuity-immediate in the amount of $P$, and 15 times a unit increasing annuity deferred by one year, paid for 19 years. Therefore, Chris’ annuity is worth
\[
P a_{\overline{20}|i} + \frac{15}{1.04} (Ia)_{\overline{19}|i} = P a_{\overline{20}|i} + \frac{15}{1.04} \frac{\bar{a}_{\overline{19}|i} - 19 \cdot 1.04^{-19}}{0.04} = 3250,
\]
and this results in \( P \approx 116 \).
Answer B.

13. November 2005 Course FM/2 Examination, Problem No. 13, and Dr. Ostaszewski’s online exercise No. 268 posted July 3, 2010
For 10,000, Kelly purchases an annuity-immediate that pays 400 quarterly for the next 10 years. Calculate the annual nominal interest rate convertible monthly earned by Kelly’s investment.

A. 10.0%  	B. 10.3%  	C. 10.5%  	D. 10.7%  	E. 11.0%

Solution.
Since payments received by Kelly are quarterly, it will be convenient to work with a quarterly interest rate, and then switch it to the annual nominal interest rate convertible monthly that we are seeking. Let $j$ be the effective quarterly rate. Then \( 10,000 = 400 a_{\overline{40}|j} \).

and this gives \( j = 0.02524 \). But we know that \( j = \frac{i^{(4)}}{4} \), and
\[
\left(1 + \frac{i^{(4)}}{4}\right)^4 = \left(1 + \frac{j^{(12)}}{12}\right)^{12},
\]
so that
\[
i^{(12)} = 12 \cdot \left(1 + j\right)^3 - 1 \approx 0.10.
\]
Answer A.
14. November 2005 Course FM/2 Examination, Problem No. 14, and Dr. Ostaszewski's online exercise No. 269 posted July 10, 2010
Payments of $X$ are made at the beginning of each year for 20 years. These payments earn interest at the end of each year at an annual effective rate of 8%. The interest is immediately reinvested at an annual effective rate of 6%. At the end of 20 years, the accumulated value of the 20 payments and the reinvested interest is 5600. Calculate $X$.

A. 121.67  B. 123.56  C. 125.72  D. 127.18  E. 128.50

Solution.
The amount of interest earned on $X$ every year is $0.08X$, but since an additional payment arrives every year, those interest payments, deposited into the account where interest is reinvested, form an increasing annuity with payments $0.08X, 0.16X, 0.24X, \ldots, 1.6X$. The total accumulation is

$$20X + 0.08X \cdot (I_{20\%})_{20\%} = 20X + 0.08X \cdot \frac{\bar{v}_{20\%} - 20}{0.06} \approx 45.3236X.$$

Therefore,

$$X = \frac{5600}{45.3236} = 123.56.$$

Answer B.

15. November 2005 Course FM/2 Examination, Problem No. 15, and Dr. Ostaszewski’s online exercise No. 270 posted July 17, 2010
You are given the following term structure of spot interest rates:

<table>
<thead>
<tr>
<th>Term (in years)</th>
<th>Spot interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00%</td>
</tr>
<tr>
<td>2</td>
<td>5.75%</td>
</tr>
<tr>
<td>3</td>
<td>6.25%</td>
</tr>
<tr>
<td>4</td>
<td>6.50%</td>
</tr>
</tbody>
</table>

A three-year annuity-immediate will be issued a year from now with annual payments of 5000. Using the forward rates, calculate the present value of this annuity a year from now.

A. 13,094  B. 13,153  C. 13,296  D. 13,321  E. 13,401

Solution.
We will denote by $s_t$ the current spot rate for maturity $t$ and by $f_t$ the current one-year forward rate from time $t - 1$ to time $t$. Then $f_1 = s_1 = 5.00\%$, and

$$1 + f_2 = \frac{(1 + s_2)^2}{1 + s_1} = \frac{1.0575^2}{1.05},$$

$$1 + f_3 = \frac{(1 + s_3)^3}{(1 + s_2)^2} = \frac{1.0625^3}{1.0575^2}.$$
1 + f_4 = \left(1 + s_4\right)^4 \left(1 + s_3\right)^3 = \frac{1.065^4}{1.0625^3}.

The discounted value of the annuity at time 1, based on these forward rates is

\[\frac{5000}{1 + f_2} + \frac{5000}{\left(1 + f_2\right)\left(1 + f_3\right)} + \frac{5000}{\left(1 + f_2\right)\left(1 + f_3\right)\left(1 + f_4\right)} = 5000 \cdot \left(1 + s_2\right)^2 + \frac{5000 \cdot \left(1 + s_3\right)^2}{\left(1 + s_2\right)^3} + \frac{5000 \cdot \left(1 + s_4\right)^3}{\left(1 + s_2\right)\left(1 + s_3\right)\left(1 + s_4\right)^4} = \frac{5000 \cdot \left(1 + s_2\right)}{\left(1 + s_2\right)^2} + \frac{5000 \cdot \left(1 + s_3\right)^3}{\left(1 + s_3\right)^4} = \frac{5000 \cdot 1.05}{1.0575^2} + \frac{5000 \cdot 1.05}{1.0625^3} + \frac{5000 \cdot 1.05}{1.065^4} \approx 13,152.50.\]

Answer B.

16. November 2005 Course FM/2 Examination, Problem No. 16, and Dr. Ostaszewski’s online exercise No. 271 posted July 24, 2010
Dan purchases a 1000 par value 10-year bond with 9% semiannual coupons for 925. He is able to reinvest his coupon payments at a nominal rate of 7% convertible semiannually. Calculate his nominal annual yield rate convertible semiannually over the ten-year period.

A. 7.6%      B. 8.1%      C. 9.2%      D. 9.4%      E. 10.2%

Solution.
At the end of 10 years, Dan will receive the principal of 1000, plus the accumulated value of reinvested coupons equal to $45s_{20|3.5\%} \approx 1272.59$, for a total of 2272.59. Thus over 10 years, he turned his initial investment of 925 into 2272.59, and if we write $i^{(2)}$ for the nominal annual yield rate convertible semiannually then we must have

\[925 \cdot \left(1 + \frac{i^{(2)}}{2}\right)^{210} = 2272.59,\]

so that

\[i^{(2)} = 2 \cdot \left(\frac{2272.59}{925}\right)^{\frac{1}{20}} - 1 \approx 9.20\% .\]

Answer C.

17. November 2005 Course FM/2 Examination, Problem No. 17, and Dr. Ostaszewski’s online exercise No. 272 posted July 31, 2010
Theo sells a stock short with a current price of 25,000 and buys it back for $X$ at the end of
1 year. Governmental regulations require the short seller to deposit margin of 40% at the
time of the short sale. The prevailing interest rate is an 8% annual rate, and Theo earns a
25% yield on the transaction. Calculate X.

A. 19,550  B. 20,750  C. 22,500  D. 23,300  E. 24,500

Solution.
Theo makes a margin deposit of 40% of 25000, i.e., 10000, as his initial cash outlay. The
problem does not say that, but since the prevailing interest rate is given, you are supposed
to assume that he earns 8% on his margin deposit, for a return of 800. You are also
supposed to assume that he does not earn interest on the proceeds of the short sale, as this
is next to impossible in reality. You might wonder how you were supposed to know these
assumptions without them being stated in the problem. This is what I call the “original
intent theory of actuarial examinations mastery”: it is crucial for you to learn to answer
the question that the examiners intended to ask, and that is not necessarily the one that
was actually asked. Theo’s profit on the sale is 25000 – X. And he made 25% on his
initial outlay of 10000, i.e., his net is 2500. Therefore, 800 + (25000 – X) = 2500, and
from this we easily get X = 23300.
Answer D.

18. November 2005 Course FM/2 Examination, Problem No. 18, and Dr.
Ostaszewski’s online exercise No. 273 posted August 7, 2010
A loan is repaid with level annual payments based on an annual effective interest rate of
7%. The 8th payment consists of 789 of interest and 211 of principal. Calculate the
amount of interest paid in the 18th payment.

A. 415  B. 444  C. 556  D. 585  E. 612

Solution.
The level annual payment is 789 + 211 = 1000. If we denote by \( P_t \) the principal portion
of the \( t \)-th payment, then
\[
P_{18} = 1.07^{10} \cdot P_8 = 1.07^{10} \cdot 211 \approx 415.07,
\]
so that the interest portion is 1000 – 415.07 = 584.93.
Answer D.

19. November 2005 Course FM/2 Examination, Problem No. 19, and Dr.
Ostaszewski’s online exercise No. 274 posted August 14, 2010
Which of the following statements about zero-coupon bonds are true?
I. Zero-coupon bonds may be created by separating the coupon payments and redemption
values from bonds and selling each of them separately.
II. The yield rates on stripped Treasuries at any point in time provide an immediate
reading of the risk-free yield curve.
III. The interest rates on the risk-free yield curve are called forward rates.
A. I only
B. II only
C. III only
D. I, II, and III
E. The correct answer is not given by A, B, C, or D.

Solution.
The correct answer is D, all statements are true. The official answer was E, because I and II are true, but III is interpreted to mean that:
- The “yield curve” in III is not the forward curve, but the spot curve,
- The forward rates implied by the spot curve are not “on” it, but determined by it.
Answer E.

20. November 2005 Course FM/2 Examination, Problem No. 20, and Dr. Ostaszewski’s online exercise No. 275 posted August 21, 2010
The dividends of a common stock are expected to be 1 at the end of each of the next 5 years and 2 for each of the following 5 years. The dividends are expected to grow at a fixed rate of 2% per year thereafter. Assume an annual effective interest rate of 6%.
Calculate the price of this stock using the dividend discount model.

A. 29       B. 33       C. 37       D. 39       E. 41

Solution.
At time 10 in the future, the price of this stock will be given by the standard dividend-discount formula as \[
\frac{2 \cdot 1.02}{0.06 - 0.02} = 51.
\]
The price at time 0 will be the present value of that 51, plus the present value of the dividends paid during the first ten years, i.e.,

\[
\frac{51}{1.06^{10}} + \frac{2a_{5\%}}{1.06^5} \approx 39.
\]
Answer D.

21. November 2005 Course FM/2 Examination, Problem No. 21, and Dr. Ostaszewski’s online exercise No. 276 posted August 28, 2010
Which of the following statements about immunization strategies are true?
I. To achieve immunization, the convexity of the assets must equal the convexity of the liabilities.
II. The full immunization technique is designed to work for any change in the interest rate.
III. The theory of immunization was developed to protect against adverse effects created by changes in interest rates.

A. None       B. I and II only       C. I and III only       D. II and III only

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E. The correct answer is not given by A, B, C, and D.

Solution.
I is false: convexity of the assets is supposed to exceed that of the liabilities.
II is sort of theoretically true, of course false in practice, because of the need for rebalancing, and immunization is not really supposed to work for large changes in interest rates, only for small ones. But if you get the convexity right, and use it in the context in which it is defined, full immunization theoretically works this way, so you were supposed to say II is true.
III is exactly true.
Answer D.

22. November 2005 Course FM/2 Examination, Problem No. 22, and Dr. Ostaszewski’s online exercise No. 277 posted September 4, 2010
A 1000 par value bond with coupons at 9% payable semiannually was called for 1100 prior to maturity. The bond was bought for 918 immediately after a coupon payment and was held to call. The nominal yield rate convertible semiannually was 10%. Calculate the number of years the bond was held.

A. 10 B. 25 C. 39 D. 49 E. 54

Solution.
The yield earned was 5% per 6 months. Let \( n \) be the number of six-month periods that this bond was held. Then
\[
45a_{\frac{5}{2}\%} + 1100 \cdot 1.05^{-n} = 918,
\]
and using a financial calculator (In BA II Plus Pro we enter \( N = 10 \), \( PV = 918 \), \( PMT = 45 \), \( FV = -1100 \), \( I = 5\% \), and then solve by entering CPT \( N \) we obtain \( n = 49.35 \), so that the number of years the bond was held is approximately half that, i.e., 24.7.
Answer B.

23. November 2005 Course FM/2 Examination, Problem No. 23, and Dr. Ostaszewski’s online exercise No. 278 posted September 11, 2010
The present value of a 25-year annuity-immediate with a first payment of 2500 and decreasing by 100 each year thereafter is \( X \). Assuming an annual effective interest rate of 10\%, calculate \( X \).

A. 11,346 B. 13,615 C. 15,923 D. 17,396 E. 18,112

Solution.
\[
X = 100 \cdot (Da)_{25\%10\%} = 100 \cdot \frac{25 - a_{25\%10\%}}{0.10} \approx 15,922.96.
\]
Answer C.
24. November 2005 Course FM/2 Examination, Problem No. 24, and Dr. Ostaszewski’s online exercise No. 279 posted September 18, 2010
A 30-year bond with a par value of 1000 and 12% coupons payable quarterly is selling at 850. Calculate the annual nominal yield rate convertible quarterly.

A. 3.5%    B. 7.1%    C. 14.2%    D. 14.9%    E. 15.4%

Solution.
It is most convenient to find the effective quarterly interest rate first. Let us call it \( j \). Note that \( j = \frac{i^{(4)}}{4} = (1+i)^{\frac{1}{4}} - 1 \). We have
\[
30a_{30j} + 1000 \cdot (1+j)^{-120} = 850.
\]
Using a financial calculator we get \( j = 3.54\% \). The nominal annual yield rate convertible quarterly is \( i^{(4)} = 4 \cdot j = 4 \cdot 3.54\% = 14.16\% \).
Answer C.

25. November 2005 Course FM/2 Examination, Problem No. 25, and Dr. Ostaszewski’s online exercise No. 280 posted September 25, 2010
The parents of three children, ages 1, 3, and 6, wish to set up a trust fund that will pay \( X \) to each child upon attainment of age 18, and \( Y \) to each child upon attainment of age 21. They will establish the trust fund with a single investment of \( Z \). Which of the following is the correct equation of value for \( Z \)?

A. \( \frac{X}{v^{17} + v^{15} + v^{12}} + \frac{Y}{v^{20} + v^{18} + v^{15}} \)
B. \( 3 \cdot (Xv^{18} + Yv^{21}) \)
C. \( 3 \cdot Xv^3 + Y \cdot (v^{20} + v^{18} + v^{15}) \)
D. \( (X+Y) \cdot \frac{v^{20} + v^{18} + v^{15}}{v^3} \)
E. \( X \cdot (v^{17} + v^{15} + v^{12}) + Y \cdot (v^{20} + v^{18} + v^{15}) \)

Solution.
The amount needed for the 1-year old is \( Xv^{17} + Yv^{20} \). The amount needed for the 3-year old is \( Xv^{15} + Yv^{18} \). And the amount needed for the 6-year old is \( Xv^{12} + Yv^{15} \). The total of these three is
\[
X \cdot (v^{17} + v^{15} + v^{12}) + Y \cdot (v^{20} + v^{18} + v^{15}).
\]
Answer E.