A perpetuity costs 77.1 and makes annual payments at the end of the year. This perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ..., \( n \) at the end of year \( (n+1) \). After year \( (n+1) \), the payments remain constant at \( n \). The annual effective interest rate is 10.5%. Calculate \( n \).

A. 17   B. 18   C. 19   D. 20   E. 21

Solution.
Compare this perpetuity to a regular increasing perpetuity starting at time 2. They would make the same payments at times 2, 3, ..., \( (n+1) \), but then at time \( (n+2) \) the regular increasing perpetuity pays \( (n+1) \), while this perpetuity pays \( n \), resulting in a difference of 1 between two payments, at time \( (n+3) \) the regular perpetuity pays \( (n+2) \), while this one pays resulting in a difference of 2 between two payments, etc. Hence the differences between the two perpetuities, the regular one and this one, both starting at time 2, amount to an increasing perpetuity starting at time \( (n+2) \). This means that the perpetuity we are considering can be viewed as a regular increasing perpetuity (i.e., a perpetuity paying 1, 2, 3, etc.), with first payment at time 2, from which an identical increasing perpetuity, with first payment at time \( (n+2) \), is subtracted. The cost of such difference of two perpetuities is

\[
\frac{v^2}{d^2} - v^{n+2} \cdot \frac{1}{d^2} = \frac{v^2}{d^2} \cdot (1 - v^n) = \frac{i^2 \cdot v^2}{d^2} \cdot \left( \frac{1 - v^n}{i} \cdot \frac{1}{i} \right) = \frac{a_n}{i}.
\]

Since \( i = 10.5\% \), we have

\[
\frac{a_n}{i} = \frac{a_n}{0.105} = 77.10,
\]
and this tells us that \( a_{n|0.5\%} = 8.0955 \). Using a financial calculator we conclude that \( n = 19 \).
Answer C.

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