Fall 2003 Society of Actuaries Course 3 Examination, Problem No. 18, also Study Note MLC-09-08, Problem No. 13

A population has 30% who are smokers with a constant force of mortality 0.2 and 70% who are non-smokers with a constant force of mortality 0.1. Calculate the 75th percentile of the distribution of the future lifetime of an individual selected at random from this population.

A. 10.7  B. 11.0  C. 11.2  D. 11.6  E. 11.8

Solution.
The survival function of this distribution is:

\[ s_X(x) = 0.30 \cdot e^{-0.2x} + 0.70 \cdot e^{-0.1x}. \]

Therefore, the 75-th percentile is given by \( x \) such that probability of survival beyond that age is 0.25:

\[ 0.25 = 0.30 \cdot e^{-0.2x} + 0.70 \cdot e^{-0.1x}. \]

If we set \( e^{-0.1x} = z \) we obtain a quadratic equation

\[ 0.3z^2 + 0.7z - 0.25 = 0. \]

This solves to

\[ z = \frac{-0.7 \pm \sqrt{0.7^2 - 4 \cdot 0.3 \cdot (-0.25)}}{2 \cdot 0.3} \approx \begin{cases} 0.31469907, \\ -2.6480324. \end{cases} \]

Therefore \( e^{-0.1x} \approx 0.31469907 \) and

\[ x \approx \frac{\ln 0.31469907}{-0.1} \approx 11.5613843. \]

Answer D.
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