
Overview of risk-adjusted performance measurement

Risk-adjusted measures take into account inherent business risks, with return on capital generally defined as Income/Capital. Adjustments can be made to either numerator or denominator. Traditional measures that do not incorporate risk: ROE, ROA, Total Shareholder Return, are misleading. Approaches to incorporating risk:

- Allocate surplus by business units based on judgment,
- Adjust return for risk (RAROC),
- Adjust capital for risk (key in this paper),
- Adjust both.

Risk-Adjusted Return on Capital (RAROC) = Income/(Risk-Adjusted Capital)

Income measures

- GAAP net income, convenient for management decision making, standard for accounting purposes.

- Statutory net income.

- International Accounting Standards Board (fair value basis) net income. Differences from GAAP: discounting of loss reserves, inclusion of a risk charge to approximate that included in a third-party transaction.

- Economic profit: change in economic value of (Assets minus Liabilities), where asset values are market values and liabilities are discounted to reflect their present value. Liabilities may include a risk margin. Should include franchise value, i.e., value of future profits. Reconciliation to GAAP difficult, especially since managers may need to communicate results to external parties who may have access only to GAAP and SAP statements.

Capital Measures

Traditional measures that are not risk adjusted:

- Actual committed capital, cash capital provided to the company by its owners. This equals contributed capital plus retained earnings. Can be based on GAAP, SAP, or IASB.

- Market value of equity, including market value adjustments to assets and liabilities, and franchise value. If franchise value is included, value is larger than committed capital.

Measures that are risk adjusted:

- Regulatory required capital, in the U.S.: Risk-Based Capital.
- Rating agency required capital: S&P, A.M. Best, Moody's, or Fitch. Calculated
using the agency's capital model.
- Economic capital: capital required to ensure a specified probability that the firm can achieve its specified objective over a given time horizon. Possible objectives:
  - Solvency: holding sufficient capital to meet existing obligations to policyholders and maybe debtholders. It is commonly used and adopted in the paper. It emphasizes cash and marketable securities. It involves an arbitrary separation of assets in liability and capital portions.
  - Capital adequacy objective: holding sufficient capital to continue to pay dividends, support premium growth, or maintain sufficient financial strength to maximize franchise value.
- Possible definitions of economic capital: Difference between assets and one of the following:
  - Undiscounted value of the expected liability, P/C SAP.
  - Discounted value of the expected liability.
  - Fair value of the expected liability, i.e., discounted value of the expected liability with interest rate increased by a risk margin, hard to account for all risks, as some are not shown in the balance sheet.
- Risk capital: amount of capital needed to absorb the risk that liabilities will exceed the funds already provided for by loss reserves or premiums. Reduced by any conservatism in reserving or risk margins, but if there were no risk margins in pricing or reserving, it may be identical to economic capital.

Are Risk-Based Capital Measures Superior?
Which capital measure is best? Risk-based ones? Risk-based measures are typically lower than a firm's market or book value. This results in unallocated capital. One can compensate for this unallocated capital by using stranded capital. Stranded capital: charges that adjust the return measure to reflect a cost associated with the actual capital held in excess of the risk-based capital, this results in allocating all capital but with risk-based adjustment.

**Measuring risk capital**

Possible Risk Measures
- Probability of ruin. Ruin usually defined as default but can be defined as a decline in credit rating.
- Percentile Risk Measure (Value at Risk): dollar amount of capital required to achieve a specific probability of ruin target. Example: 99-th percentile of loss amount distribution is capital needed to limit probability of ruin to 1%.
  Same as VaR, but two differences:
    - VaR considers market value, this methodology considers nominal amount of cash flows.
    - VaR could mean amount by which percentile deviates form the mean, or amount by which percentile falls below zero, this last definition used in this reading.
  - Conditional Tail expectation (CTE), or Tail VaR (TVaR), or Tail Conditional Expectation (TCE): mean loss for losses that exceed a specified percentile. Does not have a clear connection to amount of capital needed: capital equal to $x\%$ of CTE means that default probability is less than $1 - x\%$.  

- Expected Policyholder Deficit (EPD): mean value of the shortfall between the assets and liabilities (but only shortfall, excesses not included in calculation). Risk capital determined by multiplying the expected liability amount by the target ratio of EPD to expected liabilities. Iterative process is needed when simulation used as EPD depends on total assets, which include risk capital.

**Risk Measurement Threshold**

All risk measures above have a threshold for risk. How do we select the threshold? Possibilities:

- Bond default probabilities at a selected credit rating level. Credit rating selected in line with firm's strategy. Make the threshold consistent with the default probability for chosen rating. Two possibilities: default probability assuming firm is in a runoff situation immediately or at some specified time, or probability of being downgraded over a specified time horizon. For the runoff approach many firms use estimates of default probabilities from Bank of America’s implementation of RAROC, with 0.03% probability of default common. Some things to consider:

  - Historical vs. current estimates: historical more stable, current more accurate.
  - Time horizon: Rates of default are annual default probabilities, but horizon is lifetime of the liabilities, some increase the default probability threshold for such longer-term models.

- Management risk preferences: must consider risk and reward. Difficult to achieve consistency. Management's attitudes can be inconsistent and context-specific. Shareholders prefer warning before downgrade, or distress, not default.

- Arbitrary default probability, percentile of distribution of profit/loss, EPD. Often different measures of risk for different activities.

**Risk sources**

- Market risk: potential loss in value, over the specific horizon, which results from changes in equity prices, interest rates, foreign exchange rates and other market variables. Note that in market risk modeling insurers’ horizons are longer because must hold reserve risk exposure to maturity. This reading chooses one-year horizon for market risk.

- Credit risk: potential loss in value caused by credit events, such as counterparty default, changes in counterparty credit rating or changes in credit-specific spreads. For reinsurance default has unique meaning: definition of default may need to be broader, to
take into account a credit downgrade that would lead policyholders to commute or settle, resulting in settlements for less than the full recovery amounts. Disputes (common between insurers and reinsurers) may also lead to less than 100% of potential recoveries. There is also a possibility that reinsured losses develop differently than expected, and if reinsurer defaults exposure may substantially exceed current estimate. Finally,

- Insurance underwriting risk:
  - Loss reserves on prior policy years, risk of adverse development.
  - Underwriting risk for current period policy year: should include both unearned premiums and potential losses on one year of new business.
  - Catastrophic property loss risk.
- Other risk sources are less quantifiable but should be considered: operational risks involving people, systems, and processes, or strategic risks related to competitors.

**Loss reserve risk**

Dominant risk for a P/C company. Components:

- Process risk: risk that actual results will deviate from their expected values.
- Parameter risk: risk that the actual, but unknown, expected value of the liability varies from the estimate of that expected value because of inaccurate parameter estimates in the models.
- Model risk: risk that the actual, but unknown, expected value of the liability deviates from the estimate of that expected value because of using of the wrong model.

Loss reserving involves these analyses

- Reserve estimation error. Depicted as a confidence interval.
- Reserve distribution (probability distribution). Primary interest in this reading.

Possible measures of loss reserve risk

- Mack method: Uses traditional chain ladder, useful for stress testing.
- Hodes, Feldblum, Blumsohn: Simulation of age-to-age loss development factors (LDFs) for each development period. Use of simulation may impact reliability.
- Bootstrapping method: Use a distribution of incremental paid or incurred loss amounts to produce a loss triangle for LDFs. Requires a large number of simulations.
- Zehnwirth method: Uses log incremental paid losses, identifies common trends affecting accident years, calendar years, and development periods.
- Panning econometric approach. Points out weaknesses of other methods: ad hoc measures that do not maximize goodness-of-fit, use of cumulative loss data introduces serial correlation, incorrectly assume constant variance across development periods. Panning's proposed approach uses linear regression that minimizes squared errors, applied to incremental rather than cumulative data, model each development period separately.
- Collective risk model, with frequency and severity assumptions representing values for outstanding claims, and severity distributions reflect age of outstanding claims.
- Relationship to underwriting risk: coefficient of variation for ultimate loss distributions is estimated from coefficient of variation for underwriting risk distributions for similar classes of business.

Results using the Mack Method from a 1993 paper:
- Estimate parameters of a lognormal distribution using the method of moments. Must determine if lognormal assumption is reasonable. Need to adjust parameters for their discounted values.

Ultimate liability vs. loss development during horizon
- Use of one-year horizon compatible with market and credit VaR calculations and more consistent with calendar year income measures.
- Difference between one-year and lifetime horizons significant for high-layer excess-of-loss general liability (GL) lines.
- This reading uses lifetime of liability insurance risk distributions adjusted to reflect PV of loss amounts at the end of one-year horizon as if all claims paid then.

Ceded reinsurance recoveries
- Model directly, taking into account specific nature of reinsurance agreements, this allows for consideration of credit risk.
- Model indirectly by modeling the net reserve risk, easiest approach.

**Underwriting risk** (premium risk, new business risk)
Risk that total claim and expense costs on new business written and/or earned during a specific risk assumption horizon exceed the premiums collected during the same period. This includes both renewals and business of new customers. Models: Loss ratio distributions models, frequency and severity models, inference from reserve risk models.

**Models using loss ratio distribution**
Assume a distribution of loss ratios and combine with written premium estimated deterministically or stochastically simulated. Must estimate model parameters from:
- Company historical data, but adjust for changes in claim cost trends, premium adequacy, and premium volume.
- Industry data: can replace or supplement company data.

Choice of distribution models
- Normal.
- Lognormal, but when applied to loss ratios, left tail is too heavy and right tail is too light, better results obtained by mixing two lognormals, one with a small coefficient of variation is calibrated by small losses, another one with a large coefficient of variation is calibrated by large losses.
- Gamma.

**Models using frequency and severity**
Such models more easily account for growth and more accurately accounts for inflation. Can directly reflect changes in limits and deductibles, and the impact of deductibles on claim frequency. Types of appropriate distributions:
- Frequency: Poisson, negative binomial, or a discrete distribution derived from normal.
- Severity: lognormal, gamma, exponential, beta.

To create aggregate loss distribution:
- Analytical solution.
- Numerical methods, e.g., fast Fourier transform.
- Approximations: moments of the collective risk model approximate the aggregate distribution.
  - Standard formulas
    \[
    \text{Aggregate Loss Mean} = E(N) \cdot E(S),
    \]
    \[
    \text{Aggregate Loss Variance} = E(N) \cdot \text{Var}(S) + \text{Var}(N) \cdot (E(S))^2,
    \]
    where \(N\) is the random number of claims, \(S\) - random severity for each claim.
  - Formulas with contagion parameters to reflect parameter uncertainty (Heckman and Meyers):
    \[
    \text{Aggregate Loss Mean} = E(N) \cdot E(S),
    \]
    \[
    \text{Aggregate Loss Variance} = E(N) \cdot E(S^2) \cdot (1 + b) + \text{Var}(N) \cdot (E(S))^2 \cdot (b + c + bc),
    \]
    where \(b\) is the parameter for severity shock, \(c\) is the parameter for frequency shock.
- Approximations gives great flexibility as their approaches can be adjusted for different premium volumes, expected loss ratios, attachment points, and limits, consistently across product lines. Assumption of common shocks can be added (even with different sensitivities) to produce dependency across lines.
- Aggregate loss models can be also estimated via simulation.

Models based on inference from reserve risk
Models estimate potential variability in unpaid losses after policies are written and previously unknown parameters have been determined. Adapting to pricing risk: infer unconditional models from conditional models, use the coefficient of variation for the most recent accident year.

Property-Catastrophe Risk
Shift from historical experience
- Rarity of events.
- Exposure changes.
- Severity changes.
- Cost changes.

Typical models include various modules:
- Stochastic module/hazard module: possible occurrence of events as well as location, intensity, etc.
- Damage module (vulnerability module): use exposure information to determine damage amounts.
Financial analysis module: use of policy terms and reinsurance to determine the impact on the insurer.

Risk aggregation
Correlation and dependency are different. Correlation is a measure of linear dependency. Dependency: more general measure of the degree to which different random variables depend on each other. Perfect dependency does not imply perfect correlation.

Measures of dependency
- Empirical analysis of historical data. But data may not exist. Produces correlation estimates that are unreliable and inconsistent. Data may have little relevance for tail events, as dependency may exist under extreme events but not normal conditions.
- Subjective estimates. Problem that the number of correlation/dependency assumptions grows exponentially with the number of unique risk categories or lines of business. There are also issues of consistency.
- Explicit factor models. These link variability of different assets or asset classes to common factors. Example: use of common frequency and severity shocks.

Aggregate risk distribution
Need to combine market, credit, loss reserve, underwriting, and property-catastrophe distributions into an aggregate distribution. Techniques:
- Closed form solutions - wide variety of stand-alone distributions may make this approach impractical.
- Approximation methods. Assume all distributions are normal or lognormal, then derive model parameters from actual moments or percentiles.
- Simulation methods. Run-time and stability concerns. Method of reflecting dependency across variables and uniqueness of marginal distributions:
  - Iman-Conover method - aggregate distribution preserves rank correlation.
  - Copula method. Attempts to ensure that when variables are highly and positively correlated, both will have values in the far right (left) tail at the same time. Copulas represent correlated/dependent percentiles, reflecting values from 0 to 1. Can use a variety of distributions for the copulas: normal copula uses multivariate normal distribution, while correlated $t$'s (Student's) produce more tail dependency than normal.

Alternative approach - aggregate risk measure
NAIC square root rule for the RBC formula

$$C = \sqrt{\sum C_i^2 + \sum \sum p_{ij} C_i C_j}$$

This approach is exact when risk measure is proportional to standard deviation, and all risk distributions are normal. But only a crude approximation when conditions not met.
Aggregate risk capital and allocation to business units
Assumptions for a hypothetical case, Sample Insurance Company (SIC)
  o Invested assets - $19.6 million. Rate of return follows a normal distribution and is uncorrelated with all other risk categories.
  o Loss reserves - $19.6 million of undiscounted reserves. Reserve risk distribution follows a lognormal distribution.
  o Written premium: two lines follow different lognormal distributions.
  o Expense ratio: 5.0% of the written and earned premium, paid at the beginning of the year.
  o Simplifications – the following risks are ignored: credit risk, property-catastrophe risk, operational and strategic risks.

Aggregate risk and risk capital
  o Aggregate risk distribution simulated from the stand-alone distributions, normal copula used to account for the desired correlation/dependency among the risk types. Lognormal distribution is fitted to the resulting empirical aggregate distribution using the method of moments. Losses represented as positive amounts and profits as negative amounts. Lognormal shifted by the amount of the maximum profit to determine parameters and then shifted back for the final outcome.
  o Model combines both investment and underwriting results. 99th percentile of losses selected as the risk capital target: $8.96 million.

Allocation of Risk Capital
Methods discussed:
  o Proportional allocation based on a risk measure. Allocation based on separate risk measures for each risk source (market risk, reserve risk, underwriting risk for each line), or possibly business unit, proportional to ratio of a source's risk measures to the sum of all risk measures. Allocation basis may differ from the basis used to measure aggregate risk capital (i.e., could use VaR for one, CTE for another, or use different percentiles). Similar results less likely when highly skewed risks (e.g., property-catastrophe risk) included, but use measures that are not tail based.
  o Incremental allocation. Allocation based on the impact each risk source (business unit) has on the aggregate risk measure. Recalculate risk measure after removal of one of the risk sources: difference versus previous total equals the incremental capital allocation for the removed source. However, the sum of incremental amounts does not necessarily equal the total capital, so one might need to adjust the incremental amounts proportionately.
  o Marginal allocation - allocation based on the impacts of a small change in the risk exposure for each risk source (business unit). Instead of eliminating all of a risk source, one can eliminate one dollar of expected revenue or one dollar of expected loss; then as with the incremental method adjust so that equals total capital. This is impractical when risk sources cannot be represented relative to revenue or expected loss or marginal impacts cannot be easily determined.
  o Myers-Read method. Type of marginal allocation based on the marginal impact each risk source (business unit) has on the value of a put option on the insurance
company’s assets. Put option created by the fact that owners have option to default on losses that exceed assets and shift liability for such losses onto policyholders.

Steps in the method:
- Calculate value of default option based on current capital and current exposures.
- For each source, increase the exposure and determine the capital needed to maintain the same value for a default option, thus obtaining the capital-to-loss ratio.
- Apply the capital-to-loss ratio to expected losses to find allocated capital.

In this process, market risk is taken into account, but not given specific allocated capital. Capital is allocated only to risk sources with liabilities. One could indeed argue that it does not make sense to invest capital to protect against risk associated with marketable securities.

This method is additive: allocations to individual sources/business units add up to the total requirement when the same risk measure is used.

Some observations about this method:
- Not developed for determining risk-adjusted capital requirements.
- Calculation of the value of the default option may be quite a quantitative challenge.
- Assumes that loss distributions exhibit homogeneity, i.e. risk exposure in a business unit can be increased or decreased without impacting the shape of the loss distribution.
- Comeasures approach (Kreps, Ruhm-Mango). Allocation based on the contributions of each risk source/business unit to the aggregate risk measure. This is how it is done: Use a specific conditional risk measure to calculate the capital requirement for the entire firm and then calculate the measure for each risk source subject to the condition applied to the entire firm. For example: calculate the average losses for each risk source given that total losses exceed a chosen percentile rather than the average losses for each risk source that exceed the chosen percentile.

**Guiding strategic decisions**

**Assessing Capital Adequacy**
- Government regulators require minimum capital levels to provide protection to policyholders.
- Rating agencies assess capital and assign a financial strength or claims paying rating based on questions such as:
  - Is the firm sufficiently capitalized?
  - Does management understand sources of risk?
  - Does management measure and manage its risk exposure?
- In both cases, firms that calculate an appropriate risk measure are in better positions to demonstrate their claims paying ability.

**Setting Risk Management Priorities**
Capital allocation to business unit or activity is not required for assessing of overall capital adequacy of a firm. But allocation of capital to units/activities may offer great opportunity for managing, mitigating, or transferring risk.

Evaluating Alternative Reinsurance Strategies
RAROC measures can be used to evaluate strategies for reducing risk, and reinsurance is one of them. This can be done for the whole firm or a particular business unit.

Risk-Adjusted Performance Measurement
It is common to use historical loss ratios for comparing business units. This is misleading because those units do not necessarily have the same level of risk. Proper comparison should incorporate risk. Explicit risk adjustments are an improvement over judgmental premium-to-surplus ratios. But risk-adjustment measures are not unconditionally superior as results will vary with the method used for capital allocation.

Insurance Policy Pricing
RAROC can be used in insurance pricing, by setting price so that expected RAROC exceeds a specified target rate. Target RAROC can be calculated as:

\[
\text{Target RAROC} = \frac{\text{Original Premium} + \pi - \text{Expenses})(1 + \text{EII}) - \text{PV(Expected Claims)}}{\text{Allocated Risk Capital}}
\]

\(\pi\) = additional risk margin, EII - expected investment income. This does not include adjustment for commissions or taxes, but for a fully realistic model one should adjust for those items as related to total premium.

Additional risk margin required

\[
\text{ARMR} = \frac{\text{Alotted Risk Capital}}{\text{1 + EII}}(\text{Target RAROC} - \text{Current Economic Profit})
\]

This calculation assumes additional risk margin earns the same rate of return as net premiums. If we use the risk-free rate instead, difference is not substantial.

Some additional considerations
- Investment income on allocated capita: if capital is committed over multiple periods, it may be easier to measure economic profit at the beginning rather than the end of the year. In that case, RAROC target represents excess return over the investment return.
- Multiperiod capital commitment. We must realize that initial capital required for a policy is not the total capital cost as risk capital may be needed in subsequent periods. One can assume an average pattern (or some other typical) for the release of risk capital and use it to adjust the RAROC ratio or modify the target rate. Possible patterns for the release of risk capital:
  o Capital released at the same rate claims are paid.
  o Capital released faster than claims are paid.
  o Capital released more slowly than claims are paid.
- Based on the pattern of risk capital held, we can calculate PV cost of capital by multiplying by the cost per unit of risk capital and then discounting.
Adjusted RAROC

\[
\text{Economic Profit} = \sum \frac{R \cdot C_j}{(1+r)^t},
\]

where:
\( R \) = constant cost of risk capital each period,
\( C_j \) = beginning risk capital for each period,
\( r \) = investment income rate expected to be earned on risk capital.

Should investment income or risk-free rate be used for discounting? Note also that corporate income taxes are ignored in the above formulas.

Steady state assumption

In the long run, the company will end up holding reserves permanently, and this also requires risk capital. Instead of using PV of all future capital commitments for underwriting alone, one should use initial capital requirements for both underwriting and reserve risk. Must identify reserve risk for each line. For new lines there may be no reserve risk. Diversification affects the various formulas differently, and standard formulas above may do a poor job in approximating RAROC for such steady state case.

Cost of risk capital

- Standard models assumed: CAPM or Fama-French three-factor model.
- Need to take into consideration how RAROC is defined, as there are different definitions of risk. CAPM focuses on systematic risk, an investment's marginal contribution to the existing portfolio, while RAROC measure of risk involves the relationship between the expected value of a cash flow and certain tail values. RAROC is artificially leveraged: denominator is neither the total market value of invested capital nor committed capital (committed capital is defined as capital that could be exposed to loss). Value of target income depends on the total market value of a firm, including (hard to estimate) franchise value, rather than the (smaller) book value. Formula artificially overstates the rate of return, although there is some offset by the exclusion of changes in a firm's franchise value. Note that in a given firm unit its losses are not limited to capital allocated, which makes the leverage effect on RAROC more pronounced.

Responses to the definition and leverage issues

- Adjust CAPM/Fama-French return upwards by the ratio of a firm's total capital to the firm's risk capital as CAPM/Fama-French returns based on total capital whereas only risk capital is at risk.
- Continue to use total capital but use allocation method reflecting relative risks This fails to account for differential degrees of leverage in business units. Also fails to account for different degree of systematic risk in business units.
Alternative methods
- Feldblum incorporates frictional costs of holding capital such as the double taxation of investment income.
- Mango argues that a business unit needs to earn adequate profits to compensate for both underwriting capacity explicitly allocated and potential capital call if allocated capital is insufficient.

Practical considerations
Time Horizons
Consideration for different time horizons for market risk (one year) and insurance risks (potential ultimate liability) may be material. What to do?
- Measure market and credit risks over the entire lifetime of the insurance liabilities. This requires complex dynamic financial analysis (DFA) models. It is difficult to quantify equity market, interest rate, and foreign exchange rate parameters over long time horizons as historical data may not be available, or appropriate, and there are significant serial correlations. Over time, investment portfolio may change a lot.
- Focus on the change in the value of insurance liabilities over a one-year period, to match asset time horizon. But relevant information may not be available in the short run and thus we may show little change in value even when substantial risk exists. Example: higher layer excess general liability insurance in Europe, because of Solvency II rules, has moved towards measuring insurance risks over short time horizon.

Alternative Risk Measures
- Accounting measures: Because of familiarity with GAAP or SAP, some firms continue to use them in calculating RAROC, producing something like a calendar year RAROC.
- The measure should include taxes, and this gets complicated.
- Stranded capital: recall that this is the amount of capital held in excess of the (allocated) risk capital. To account for the leverage effect, one should reduce RAROC return measures for the cost of stranded capital. Possible definitions of capital held: capital required by regulators, or firm’s total capital. Reducing the rate of return is equivalent to adjustment involving allocation of the firm’s total capital rather than just the firm’s risk capital.
- Investment income: If working with multiple periods, it is easier to use present values but this makes it more difficult to include investment income, and resulting RAROC is not a true rate of return.

Risk-Based Allocation
- Allocating risk capital based on tail measures appropriate for regulatory or rating agency applications, but not so for management or shareholder interests.
- To take into account shareholder interests:
Use measures that reflect events affecting a firm's credit rating, financial strength, or claims paying ability and allocate capital based on those measures.

- Use tail measures of risk but different percentiles.

*Diversification Adjustments*
RAROC is informative but it is not the only metric driving decisions regarding a firm's risk management. Correlation/dependency across business units and risk sources is also important.

*Appendices*

*Risk Exposure Horizon*
This is the time period over which the risk can affect the company. But models and data are usually annual. Responses to problem of consistency of the risk horizon exposure:

- Use one-year risk exposure horizon for all risk types. This may be difficult: hard to know exactly when adverse loss development is recognized, also firms differ in reporting practices, as well as in whether test adequacy is done by line or in the aggregate. Furthermore, perspective that focuses on best estimates not consistent with inclusion of risk margin, which is used market value analysis. Finally, risk of loss reserves not resolved over a single period.

- Use multi-period DFA models: this involves complexity beyond extending one-year metrics because of model parameters estimation, serial correlation, and possible changes in management strategy.

- Ignore the inconsistencies, and measure some risks over a one-year period and others over the lifetime of the risk. If two approaches are aggregated, models become difficult to interpret. An alternative is to not explicitly aggregate market and credit risks with insurance risks but rather reduce current asset balances to reflect potential loss in value over the year, this is approach followed in Standard and Poor’s capital adequacy ratio (CAR).

*Myers-Read Capital Allocation*
Firm's default option: right to default in whole or in part on their obligations to policyholders. Two possible cases:

- Assets fixed and liabilities risky, call option is on liabilities with a strike price equal to assets.

- Assets and liabilities both risky, option is to exchange assets for liabilities. This case cannot be easily quantified by the Black-Scholes pricing formula. Cummins characterizes it as a standard put option on the asset/liability ratio with a strike price of 1. Ratio falling below 1 indicates insolvency. Volatility in Black-Scholes put formula is the volatility of the asset/liability ratio, and its calculation involves volatilities of assets and liabilities, as well as their correlations. In case of independence of assets and liabilities

\[
\sigma_{\text{Assets/Liabilities}}^2 = \sigma_{\text{Assets}}^2 + \sigma_{\text{Liabilities}}^2
\]

The volatilities of assets and liabilities above are volatilities of geometric Brownian motions that assets and liabilities are assumed to follow.
Myers-Read allocation
Marginal capital for a unit/product line reflects the effect on the default option value of a small increase in the expected losses for the line. Capital determined such that each line has the same marginal impact on the overall default option value. This does not give total capital amount required but only how it should be allocated. Cummins assumes 5% EPD ratio target for the whole firm (default option worth 5% of expected aggregate losses) and the resulting capital-to-liability ratio for line $i$ is

$$s_i = s - \frac{\partial \sigma}{\partial s} \left( (\sigma_{IL} - \sigma_{LV}^2) - (\sigma_{LV} - \sigma_{IL}) \right) \cdot \frac{1}{\sigma}$$

$s_i$ = capital-to-liability ratio for line $I$,
$s$ = capital-to-liability ratio for all lines combined,
$\sigma_{IL}$ = covariance of line $i$ losses with total losses for all lines,
$\sigma_{LV}$ = covariance of line $i$ losses with the assets,
$p$ = value of firm's insolvency put,
$\sigma$ = overall volatility of the assets-to-liability ratio.

Note also that

$$\frac{A}{L} = 1 + s,$$

$$\sigma_L^2 = \sum w_i^2 \cdot \sigma_j^2 + \sum \sum w_i \cdot w_j \cdot \sigma_{ij} = \sum w_i \cdot \sigma_{il},$$

$$p = N(-d_2) - (1 + s)N(-d_1),$$

where $d_1 = \frac{\ln(1 + s)}{\sigma} + \frac{\sigma}{2}$, $d_2 = d_1 - \sigma$.

From this

Option delta: $\frac{\partial p}{\partial s} = -N(d_1)$, Option vega $\frac{\partial p}{\partial \sigma} = N'(d_2)$.

May 2007 Casualty Actuarial Society Course 8 Examination, Problem No. 35

The CEO of your company has asked you for assistance in determining which lines of business written by the company are value adding and which are value detracting, as evaluated on a rate-of-return basis.

a. Briefly describe four possible measures of capital that could be used in the denominator of the rate-of-return calculation.

b. Briefly explain how your recommended capital measure might differ based on whether your company is publicly owned or is a mutual company.

c. Briefly describe two possible risk measures that could be used to assess the risk associated with a given line of business.
d. Briefly describe three methods to allocate capital to a given line of business once the appropriate risk measure has been selected.

e. The CEO would like to use the company's cost of capital, as derived from the capital asset pricing model, as the hurdle rate to which each line's rate of return should be compared. Explain why this may or may not be appropriate.

Solution.

a. **Actual committed capital**: capital provided to the company by its shareholders and used to generate income for the firm and its respective business units.

*Market value of equity*: equity measure either based on the adjustment of assets and liabilities to market value or equal to the market value of the firm's equity, which includes franchise value.

*Regulatory required capital*: capital needed to satisfy minimum requirements, usually based on a formula.

*Rating agency required capital*: capital required to achieve a stated credit rating from one or more credit rating agencies.

*Economic capital*: capital required to ensure a specified probability (level of confidence) that the firm can achieve a specified objective (involving either solvency or capital adequacy) over a given time horizon.

*Risk capital*: amount of capital that must be contributed by the shareholders of the firm in order to absorb the risk that liabilities will exceed the funds already provided for in either the loss reserves or in the policyholder premiums.

b. The objective underlying economic capital can vary depending upon whether the focus is on the policyholder, debtholder or shareholder perspectives. Public companies focus on shareholders, whereas mutual companies focus on policyholders.

c. **Probability of ruin**: Estimated probability that a ruin will occur.

*Percentile risk measure* (value at risk): dollar amount of capital required to achieve a specific probability of ruin target.

*Conditional tail expectation* (CTE) a.k.a. tail VaR (TVaR), tail conditional expectation (TCE): average loss for those losses that exceed the chosen percentile.

*Expected policyholder deficit* (EPD) ratio: Average value of the shortfall between the assets and liabilities.
d. Proportional allocation based on a risk measure: allocation based on separate risk measures for each risk source (business unit).

Incremental allocation: allocation based on the impact each risk source (business unit) has on the aggregate risk measure.

Marginal allocation: allocation based on the impacts of a small change in the risk exposure for each risk source (business unit).

Myers-Read method: type of marginal allocation based on the marginal impact each risk source (business unit) has on the value of a put option on the insurance company's assets.

Comeasures approach: allocation based on using a particular conditional measure to establish the firm-wide capital requirement and calculating comparable risk measures for each unit subject to the condition applied to the entire firm.

e. Rate of return incorporates an entirely different measure of risk based on the relationship between a cash flow's expected value and certain values in the tail of its probability distribution. Leverage varies by business unit. Risk varies by business unit.
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 35

You are given the following information about an insurance line of business:

- The expected premium is $5,000,000.
- The expected expense ratio is 5%.
- The expected annual investment return is 4%.
- The expected discounted loss ratio is 90%.
- The target risk-adjusted return on capital (RAROC) is 15%.
- The co-conditional tail expectation (Co-CTE) allocated risk capital at 99th percentile is $4,000,000.

a. (1.25 points)

Calculate the expected RAROC using the Co-CTE (99%) allocation for this line of business.

b. (1 point)

Calculate the additional risk margin in dollars required to produce the target RAROC using an economic value added (EVA) approach.

Solution.

Prem = 5M  
Expense = 5%  
investment = 4%  
LR = 90%  
Target K = .15  
allocated capital = 4M

a.  
1. Prem  5,000,000  
2. Expenses  5%(5M) = 250,000  
3. Investment [(1)-(2)] x .04 = 190,000  
4. Losses  Line (1) x (.9) = 5M(.9) = 4,500,000  
5. Income  1 – 2 + 3 – 4 = 5M - .25M + .19M – 4.5M  
  = 440,000

RAROC = \( \frac{440,000}{4,000,000} \) = 11%

b. additional risk margin:

required income = 4m (.15) = 600,000  
actual = 440,000  

\( \frac{(600,000 - 440,000)}{1.04} = 153,846 \)
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 36

Assume a monoline insurance company has $7,500,000 of capital. The company’s only three sources of risk are market risk, loss reserve risk, and underwriting risk.

These key risk components were simulated and the aggregate loss was determined for each of 1,000 simulation scenarios. The five largest simulated aggregate losses are shown in the following table.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Market Risk</th>
<th>Reserve Risk</th>
<th>Underwriting Risk</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>($950,000)</td>
<td>$3,600,000</td>
<td>$1,900,000</td>
<td>$4,550,000</td>
</tr>
<tr>
<td>2</td>
<td>250,000</td>
<td>2,750,000</td>
<td>750,000</td>
<td>3,750,000</td>
</tr>
<tr>
<td>3</td>
<td>100,000</td>
<td>1,900,000</td>
<td>950,000</td>
<td>2,950,000</td>
</tr>
<tr>
<td>4</td>
<td>(400,000)</td>
<td>1,200,000</td>
<td>1,400,000</td>
<td>2,200,000</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1,500,000</td>
<td>550,000</td>
<td>2,050,000</td>
</tr>
</tbody>
</table>

a. (0.5 point)

Calculate the 99.5% Conditional Tail Expectation (CTE) for the total aggregate loss.

b. (1.5 points)

Allocate the capital to each source of risk (market, reserve, underwriting) in proportion to the 99.5% Co-CTE allocation percentages.

Solution.

(a) \( CTE = \text{Average of Total} = 3,100,000 \)

(b)

<table>
<thead>
<tr>
<th></th>
<th>Allocated Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market CTE = -200,000</td>
<td>-483,871</td>
</tr>
<tr>
<td>Reserve CTE = 2,190,000</td>
<td>5,298,387</td>
</tr>
<tr>
<td>UIW CTE = 1,110,000</td>
<td>2,685,484</td>
</tr>
</tbody>
</table>
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 37

You are given the following information about two lines of business written by an insurer for which economic profit equals net income:

<table>
<thead>
<tr>
<th></th>
<th>Line A</th>
<th>Line B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual discounted loss ratio</td>
<td>95.0%</td>
<td>85.0%</td>
</tr>
<tr>
<td>Economic profit</td>
<td>500,000</td>
<td>900,000</td>
</tr>
<tr>
<td>Co-conditional tail expectation (99%) allocated capital</td>
<td>2,000,000</td>
<td>4,000,000</td>
</tr>
<tr>
<td>Cost of capital</td>
<td>10%</td>
<td>18%</td>
</tr>
</tbody>
</table>

a. (0.75 point)

Explain which line of business performed better based on the risk-adjusted return on capital (RAROC) performance measure.

b. (0.75 point)

Explain whether each line of business adds value for the insurer based on the economic value added (EVA) performance measure.

Solution.

a. \[ \text{Income} = \text{RAROC} \]

\[ A = \frac{500}{2,000} = 25\% \quad B = \frac{900}{4,000} = 22.5\% \]

Line A performed better since its RAROC is bigger.

b. \[ \text{EVA} A = 500,000 - 2M(.10) = 300,000 \]

\[ \text{EVA} B = 900,000 - 4M(.18) = 180,000 \]

Each line of business adds value since their EVA’s are greater than 0.
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 39

You are given the following information about insurers A and B:

- Insurers A and B both have assets of $13,000.
- Insurers A and B both have unpaid losses of $10,000.
- Insurers A and B both have capital of $3,000.

- Insurer A’s Loss Probability Distribution

<table>
<thead>
<tr>
<th>Probability</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>$6,900</td>
</tr>
<tr>
<td>60%</td>
<td>$10,000</td>
</tr>
<tr>
<td>20%</td>
<td>$13,100</td>
</tr>
</tbody>
</table>

- Insurer B’s Loss Probability Distribution

<table>
<thead>
<tr>
<th>Probability</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>$2,000</td>
</tr>
<tr>
<td>60%</td>
<td>$10,000</td>
</tr>
<tr>
<td>20%</td>
<td>$18,000</td>
</tr>
</tbody>
</table>

Demonstrate why probability of ruin is inadequate as a measure of risk with respect to solvency for insurers A and B.

Solution.

\[
\text{EPD}_A = \frac{100 \times 0.2}{E[L]} = 0.002
\]

\[
E[L] = 6.9k(0.2) + 10k(0.6) + 13.1k(0.2)
\]

\[
\text{EPD}_B = \frac{5,000 \times 0.2}{E[L]} = 0.10
\]

\[
E[L] = 2k(0.2) + 10k(0.6) + 18k(0.2)
\]

Probability of ruin is 0.2 for both insurer, both severity of ruin is much higher for B.
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 40

You plan to allocate capital to a line of business with the following characteristics:

- Both assets and liabilities are risky.
- The risk parameter is 0.5.
- The asset-to-liability ratio is 1.75.
- The time to maturity is one year.
- The risk-free rate is 1%.

Calculate the expected policyholder deficit as a percentage of liabilities.

Solution.

EPD ratio

\[ \sigma = 0.5 \]

\[ S_0 = \frac{A}{L} = 1.75 \quad K = 1 \quad T = 1 \quad rf = 1\% \]

\[ d_1 = \frac{\ln(5/k) + (rt + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln(1.75) + (1\% - 0.5^2/2)}{0.5} = 1.389 \]

\[ d_2 = d_1 = \sigma\sqrt{T} = 1.389 - 0.5 = 0.889 \]

\[ N(d_1) = 0.9177 \]

\[ N(d_2) = 0.8133 \]

\[ C = S_0N(d_1) = ke^{rT}N(d_2) \]

\[ = 1.75(0.9177) - e^{-0.01}(0.8133) = 0.801 \]

\[ P = c + ke^{rT} - S_0 \]

\[ = 0.801 + e^{-0.01} - 1.75 = 0.041 \]
Spring 2007 Casualty Actuarial Society Course 8 Examination, Problem No. 35

The CEO of your company has asked you for assistance in determining which lines of business written by the company are value-adding and which are value-detracting, as evaluated on a rate-of-return basis.

a. (1 point)

Briefly describe four possible measures of capital that could be used in the denominator of the rate-of-return calculation.

b. (0.5 point)

Briefly explain how your recommended capital measure might differ based on whether your company is publicly owned or is a mutual company.

c. (1 point)

Briefly describe two possible risk measures that could be used to assess the risk associated with a given line of business.

d. (1.5 points)

Briefly describe three methods to allocate capital to a given line of business once the appropriate risk measure has been selected.

e. (0.5 point)

The CEO would like to use the company’s cost of capital, as derived from the Capital Asset Pricing Model, as the hurdle rate to which each line’s rate of return should be compared.

Explain why this may or may not be appropriate.

Solution.
Part A – the following answers received full credit:

- Actual committed capital (or Book value of capital) – the actual cash capital provided to the company by its shareholders and used to generate income for the firm and its respective business units
- GAAP Equity – book value of capital using GAAP accounting principles
- Statutory Surplus – book value of capital using statutory accounting principles
- IASB Fair Value of capital – value of capital using IASB accounting principles
- Market value of equity – the committed capital measure adjusted to reflect market values of the assets and liabilities
- Regulatory required capital (or RBC) – the capital required to satisfy minimum regulatory requirements
- Rating agency required capital – the capital required to achieve a stated credit rating from one or more credit rating agencies (S&P, A.M. Best, Moody’s or Fitch)
- Economic capital – the capital required to ensure a specified probability (level of confidence) that the firm can achieve a specified objective over a given time horizon
- Risk capital – the amount of capital that must be contributed by the shareholders of the firm in order to absorb the risk that liabilities will exceed the funds already provided for in either the loss reserves or in the policyholder premiums

- Premium
- Assets

Part B – A publicly owned company would choose a different capital measure than a mutual due to differences in interests. Policyholder interests are more focused on solvency – paying claims. Shareholder interests are more focused on the company as a going concern and with maximizing firm value. Since policyholders own the mutual company, and shareholders own the public company, each will choose a different measure in line with those interests.

Part C – the following answers received full credit:

- Probability of Ruin – the (estimated) probability that losses exceed capital
- Value at Risk – the dollar amount that will not be exceeded in the next N days with X% certainty
- Expected Policyholder Deficit – looks at the expected difference between the amount the company is obligated to pay and the amount it actually pays to the obligee
- Conditional Tail Expectation – measures the average loss of the scenarios worse than the X% percentile
- Risk Based Capital – perform the NAIC RBC calculations for each line of business
- CAPM – decompose the beta coefficient to determine the betas by line of business
Part D – the following answers received full credit:

- Proportional allocation based on a risk measure – calculates standalone risk measures for each risk source and then allocates the total risk capital in proportion to the separate risk measures.
- Incremental Allocation – using a risk measure, measure the capital needed by the total firm and the capital for the firm excluding a single line of business. The capital allocation can either be the difference between these two numbers (Merton & Perold) or a proportional allocation based on the distribution of the differences for all lines of business.
- Marginal Allocation – using a risk measure, measure the capital needed by the total firm and the capital for the firm after making a small change in the risk exposure for a risk source (e.g. amount of assets, amount of reserves, premium volume). Allocate the total capital in proportion to these marginal changes in capital. (Myers & Read is an example of this).
- Co-Measures approach – establish the capital requirement using a particular conditional risk measure, such as VaR or CTE, and then calculate the Co-Measure for each business unit by calculating the comparable risk measure for the unit subject to the condition applied to the entire firm.
- Regulatory Risk-based Capital (RBC) – perform the NAIC RBC calculations for each line of business
- CAPM – decompose the beta coefficient to determine the betas by line of business
- Insolvency Put Option – model the policyholders’ claim on the company’s assets as a put option on the ratio of assets to liabilities with a strike price of 1.00

Part E – the following answers received full credit:

- The CAPM cost of capital is based on the market value of the company’s equity. The risk capital used in the rate of return calculations is most likely not market value of equity. Therefore, some sort of adjustment is likely needed.
- Each line of business has different amounts of risks and different amounts of surplus supporting it. Hence the same rate for all lines is not appropriate.
Spring 2007 Casualty Actuarial Society Course 8 Examination, Problem No. 36

You are given the following information:

- Assets = $1,000
- Liabilities = $1,196
- Risk-free rate (continuously compounded) = 6%
- Time to payment = 1 year
- Risk parameter (continuously compounded) = 10%

Calculate the Expected Policyholder Deficit.

Solution.

EPD ratio = expected shortfall / loss = put option / loss

EPD is equivalent to a put on the assets of the firm with k = PV (liabilities)

Put option

d1 = [ln (1000/1196) + (.06 + .5*.1^2)] / .1 = -1.14

d2 = -1.14 - .1 = -1.24

N(d1) = .1271
N(d2) = .1075

c = 1000 * .1271 - 1126.35 * .1075 = 6.02

put-call parity → p = 6.02 + 1126.35 - 1000 = 132.37