Below you will find exercises on topics of MAT 483, written as multiple choice problems, with solutions. Please use these exercises in preparation for the final examinations. Additionally, please study carefully previous tests given in the class, study notes provided, and please note links to past SOA and CAS exams in the class syllabus. In addition to the problems already given in the exercises below, and class notes and old tests, also study these problems (note that solutions are provided with these exams):

- Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 13
- Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 27
- Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 28
- Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 30
- Spring 2001 Casualty Actuarial Society Course 8 Examination, Problem No. 29
- Spring 2001 Casualty Actuarial Society Course 8 Examination, Problem No. 31
- Spring 2001 Casualty Actuarial Society Course 8 Examination, Problem No. 47
- Spring 2002 Casualty Actuarial Society Course 8 Examination, Problem No. 7
- Spring 2002 Casualty Actuarial Society Course 8 Examination, Problem No. 8
- Spring 2002 Casualty Actuarial Society Course 8 Examination, Problem No. 17
- Spring 2002 Casualty Actuarial Society Course 8 Examination, Problem No. 24
- Spring 2002 Casualty Actuarial Society Course 8 Examination, Problem No. 27
- Spring 2002 Casualty Actuarial Society Course 8 Examination, Problem No. 38
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 3
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 4
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 5
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 6
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 7
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 10
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 12
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 13
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 18
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 22
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 25
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 26
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 27
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 32
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 34
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 37
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 38
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 39
- Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 43
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 3
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 6
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 8
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 9
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 13
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 14
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 15
- Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 17
Exercise
You are the investment actuary for a life insurance company that has sold a Guaranteed Investment Contract (GIC) to a pension plan. The GIC promises to pay exactly 5% annual effective interest rate on a deposit of $1,000,000 for the next five years. The entire principal and interest are paid to the pension plan upon maturity of the GIC in five years exactly, and no other payments are to be made under any circumstances. Upon receipt of the $1,000,000 deposit from the pension plan, the funds are invested by the life insurance company in a portfolio consisting of a one-year Treasury Bill, five-year zero-coupon Treasury Note, and a ten-year zero-coupon Treasury Bond. You can assume that Treasury issues are free of credit risk and prepayment risk. What is the convexity of the asset portfolio if you immunize your liabilities with assets available, and you choose to maximize the convexity of the asset portfolio? Only nonnegative positions in all three assets are allowed. Current interest rate is $i = 6\%$ for all maturities.

A. 45  B. 51  C. 60  D. 68  E. 75

Solution.
The Macaulay duration of the liabilities portfolio is 5. Its effective duration is
\[
\frac{5}{1.06} \approx 4.71698.
\]
If $w_1$ is the portion of the portfolio invested in the Treasury Bill, $w_2$ is the portion invested in the Treasury Note, and $1 - w_1 - w_2$ is the portion invested in the Treasury Bond, then the Macaulay duration of the asset portfolio is:
\[
1 \cdot w_1 + 5 \cdot w_2 + 10 \cdot (1 - w_1 - w_2),
\]
while the duration of it is:
\[
\frac{1 \cdot w_1 + 5 \cdot w_2 + 10 \cdot (1 - w_1 - w_2)}{1.06}.
\]
Matching the duration of the assets and liabilities is equivalent to matching Macaulay durations, and in this case produces the equation:
\[
1 \cdot w_1 + 5 \cdot w_2 + 10 \cdot (1 - w_1 - w_2) = 5,
\]
or
\[
5 - 9w_1 - 5w_2 = 0,
\]
so that
\[
w_2 = 1 - \frac{9}{5}w_1.
\]
Convexity of the asset portfolio is:
\[
\frac{1 \cdot 2}{1.06^2} \cdot w_1 + \frac{5 \cdot 6}{1.06^2} \cdot w_2 + \frac{10 \cdot 11}{1.06^2} \cdot (1 - w_1 - w_2) = \frac{1}{1.06^2} \cdot \left(2w_1 + 30 \left(1 - \frac{9}{5}w_1\right) + 110 \left(1 - w_1 - 1 + \frac{9}{5}w_1\right)\right) = \frac{1}{1.06^2} \cdot (2w_1 + 30 - 54w_1 + 88w_1) = \frac{1}{1.06^2} \cdot (36w_1 + 30).
\]
We need to maximize this quantity. This is equivalent to maximizing $36w_1 + 30$, but we should note that positions in three assets must be non-negative, so that:

\[
\begin{align*}
  w_1 &\geq 0, \\
  w_2 &= 1 - \frac{9}{5}w_1 \geq 0,
\end{align*}
\]

and

\[
1 - w_1 - w_2 = 1 - w_1 - 1 + \frac{9}{5}w_1 = \frac{4}{5}w_1 \geq 0.
\]

The first of the three inequalities is equivalent to the third one, so we only need to concern ourselves with the second one, i.e., $1 - \frac{9}{5}w_1 \geq 0$, or $w_1 \leq \frac{5}{9}$. Therefore, we need to find the maximum value of $36w_1 + 30$ for $0 \leq w_1 \leq \frac{5}{9}$. Clearly the maximum occurs when $w_1 = \frac{5}{9}$ and it equals:

\[
\frac{1}{1.06^2} \left( 36 \cdot \frac{5}{9} + 30 \right) = \frac{50}{1.06^2} \approx 44.50.
\]

Answer A.
Exercise
Company X and Company Y both want to borrow $50 million for 5 years. They have been offered fixed and floating interest rates as shown below:

<table>
<thead>
<tr>
<th>Company</th>
<th>Fixed rates</th>
<th>Floating rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>6.0%</td>
<td>6-month LIBOR + 0.5%</td>
</tr>
<tr>
<td>Y</td>
<td>8.0%</td>
<td>6-month LIBOR + 1.5%</td>
</tr>
</tbody>
</table>

Assuming each company must use financing form (fixed or floating) for which it has comparative advantage, but because of the form of their income they prefer the opposite method of financing, and to address that they then arrange a swap between themselves in a manner that splits the benefit of relative advantage equally between them. Give the structure of the swap from the perspective of Company X, assuming that the floating payment is exactly LIBOR.

A. X receives 7.5% and pays LIBOR,
B. X receives 6% and pays LIBOR,
C. X receives LIBOR and pays 7.5%
D. X receives LIBOR and pays 6%
E. X receives LIBOR and pays 7%

Solution.
Company X pays less either way, be it fixed where it pays 200 bps less, or floating, where it pays 100 bps less, than Company Y. But X’s relative advantage is in the fixed rate financing, because it has a greater advantage there, while Y’s relative advantage is in the floating area, where its disadvantage is less. Thus X borrows at 6% fixed, and Y borrows at 6-month LIBOR plus 150 bps, and after that they swap. As a result of the swap, X will switch from fixed financing to floating, so X will receive fixed to cover its 6% payment, and pay floating, while Y will receive floating and pay fixed. We are told that in the swap the floating payment is LIBOR, so we only need to figure out the fixed swap payment. Let us write $z$ for it. Consider all financing effects of X:

- X pays fixed 6% on its loan, receives fixed payment or $z$, and pays LIBOR. Its benefit over financing at a floating rate of LIBOR + 0.5% is:
  \[(\text{LIBOR} + 0.5\%) - (\text{LIBOR} + 6\% - z) = z - 5.5\%\].
- Y pays floating at LIBOR plus 1.5%, receives LIBOR, and pays $z$. Its benefit over financing at fixed 8% is:
  \[8\% - (\text{LIBOR} + 1.5\% - \text{LIBOR} + z) = 6.5\% - z\].

For these two benefits to be equal, we must have $z = 6\%$. This gives answer B. You could also observe that X’s fixed cost of 6% represents a benefit of 200 bps versus Y’s fixed cost, and Y’s LIBOR + 150 bps represents a loss of 100 bps versus X’s floating cost. Net benefit is 100 bps, and each company’s benefit should be 50 bps. From that, we can deduce the same answer.

Answer B.
You are given the following with respect to a bond with semi-annual coupon payments priced to yield 8% nominal annual compounded semi-annually:

<table>
<thead>
<tr>
<th>Semi-Annual Payment</th>
<th>Present Value of Payment at Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period (t)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>103.00</td>
</tr>
</tbody>
</table>

Calculate the convexity of the bond with respect to $i^{(2)}$, the nominal annual interest rate compounded semiannually.

A. 1.29       B. 1.58       C. 8.78       D. 9.50       E. 17.56

Solution.
The actual exam question just asked for the convexity. But this author finds the approach of the problem and its solution problematic, although valuable for learning. Let us study this exam question a bit. It is unclear from the wording of the problem whether the yield of 8% is the effective annual interest rate or the nominal annual interest rate compounded semiannual interest rate (which is suggested by the semi-annual bond coupons). Let us examine the effective annual interest rate implied by the present values of cash flows given (note that we count time in years):

- For cash flow at time 0.5, $i = \left(\frac{3.00}{2.89}\right)^{0.50} - 1 \approx 7.76\%$. But also note that $\frac{3}{1.08^{0.50}} \approx 2.89$, so that the discrepancy between 7.76% and 8.00% can be due to rounding error.

- For cash flow at time 1, $i = \frac{3.00}{2.78} - 1 \approx 7.91\%$. But also note that $\frac{3}{1.08} \approx 2.78$, so that the discrepancy between 7.91% and 8.00% can be due to rounding error.

- For cash flow at time 1.5, $i = \left(\frac{3.00}{2.67}\right)^{\frac{1}{2}} - 1 \approx 8.08\%$. But also note that $\frac{3}{1.08^{1.5}} \approx 2.67$, so that the discrepancy between 8.08% and 8.00% can be due to rounding error.

- For cash flow at time 2, $i = \left(\frac{3.00}{2.57}\right)^{\frac{1}{2}} - 1 \approx 8.04\%$. But also note that $\frac{3}{1.08^{2}} \approx 2.57$, so that the discrepancy between 8.04% and 8.00% can be due to rounding error.

- For cash flow at time 2.5, $i = \left(\frac{3.00}{2.47}\right)^{\frac{1}{3}} - 1 \approx 8.09\%$. But also note that $\frac{3}{1.08^{2.5}} \approx 2.47$, so that the discrepancy between 8.09% and 8.00% can be due to rounding error.
• For cash flow at time 2, \( i = \left( \frac{103.00}{81.76} \right)^{\frac{1}{3}} - 1 \approx 8.00\% \). This one gives us exactly the yield.

Given this, we will proceed under the assumption that 8\% is the effective annual interest rate. Note first that the sum of present values is 95.14. The convexity with respect to the force of interest is:

\[
0.5^2 \cdot \frac{2.89}{95.14} + 1^2 \cdot \frac{2.78}{95.14} + 1.5^2 \cdot \frac{2.67}{95.14} + 2^2 \cdot \frac{2.57}{95.14} + 2.5^2 \cdot \frac{2.47}{95.14} + 3^2 \cdot \frac{81.76}{95.14} \approx 8.10455644.
\]

This is closest to answer C. But the original question asked for convexity. The Macaulay duration is

\[
0.5 \cdot \frac{2.89}{95.14} + 1 \cdot \frac{2.78}{95.14} + 1.5 \cdot \frac{2.67}{95.14} + 2 \cdot \frac{2.57}{95.14} + 2.5 \cdot \frac{2.47}{95.14} + 3 \cdot \frac{81.76}{95.14} \approx 2.78352954.
\]

Convexity with respect to the interest rate \( i \) is:

\[
C = \frac{1}{(1+i)^2} C_M + \frac{1}{(1+i)^2} D_M \approx
\]

\[
= \frac{1}{1.08^2} \cdot 8.10455644 + \frac{1}{1.08^2} \cdot 2.78352954 = 9.33477879.
\]

This is closest to answer D. This is, of course, very confusing. The solution published by the SOA did the following. It treated fractional times as whole numbers, i.e., counted time in half years, and used the basic formula for convexity for fixed cash flows

\[
C = \frac{1}{(1+i)^2} \sum w_t \cdot t(t+1),
\]

as well as 4\% as the effective semi-annual interest rate, so that it calculated convexity in half-years squared as

\[
\begin{align*}
&1 \cdot \frac{1}{1.04^2} \cdot \frac{2.89}{95.14} + 2 \cdot \frac{1}{1.04^2} \cdot \frac{2.78}{95.14} + 3 \cdot \frac{1}{1.04^2} \cdot \frac{2.67}{95.14} + 4 \cdot \frac{1}{1.04^2} \cdot \frac{2.57}{95.14} + 5 \cdot \frac{1}{1.04^2} \cdot \frac{2.47}{95.14} + 6 \cdot \frac{1}{1.04^2} \cdot \frac{81.76}{95.14} \approx 35.1195311.
\end{align*}
\]

The convexity measure in years squared is therefore \( \frac{35.1195311}{4} \approx 8.77988278 \). Division by four is caused by the fact that there are four half-years squared in one year squared. Or, you can simply observe that the calculation produces convexity with respect to \( j = \frac{i}{2} \), assuming \( j = 4\% \), and that \( \frac{d^2P}{d\left(\frac{i}{2}\right)^2} = \frac{1}{4} \frac{d^2P}{dj^2} \). This gives answer C. There is a problem with this approach. First, the convexity so calculated is with respect to the nominal annual interest rate \( i \), and it is not the regular convexity calculated with respect to the annual effective interest rate \( i \). Moreover, the semiannual effective interest rate implied by the cash flow at time 0.5 years is \( \frac{3.00}{2.89} - 1 \approx 3.81\% \), not 4\%. Also,
\[
\frac{3}{1.04} = 2.88 \neq 2.89, \text{ so that the discrepancy cannot be explained by rounding error. The semiannual rate implied by the cash flow at time 3 years is } \left( \frac{103.00}{81.76} \right)^{\frac{1}{6}} - 1 \approx 3.92\%, \text{ and } \frac{103.00}{1.04^6} \approx 81.40 \neq 81.76. \text{ The values given in the problem are inconsistent with the interest rate used in the published SOA solution. Let us ask ourselves one more question: what is the value of convexity with respect to } i^{(2)}? \text{ Convexity with respect to nominal annual interest rate } i^{(m)} \text{ is}
\]
\[
C^{(m)} = \frac{1}{\left(1 + \frac{i^{(m)}}{m}\right)^2} C_M + \frac{\left(1 + \frac{i^{(m)}}{m}\right)^2}{m} D_M.
\]

Therefore,
\[
C^{(2)} = \frac{1}{\left(1 + \frac{i^{(2)}}{2}\right)^2} C_M + \frac{\left(1 + \frac{i^{(2)}}{2}\right)^2}{\frac{1}{2}} D_M = \frac{1}{1.08} \cdot 8.10455644 + \frac{1}{1.08} \cdot \frac{1}{2} \cdot 2.78352954 \approx 8.79289001.
\]

This is closest to answer C.

Answer C.
You have purchased a 20-year bond with a yield-to-maturity of 10% and a duration of 12 years. Immediately after purchase, there is a shock to interest rates, which shifts the yield-to-maturity of the bond to 12% and the duration of the bond to 11 years. Assuming that the new interest rates persist indefinitely, determine the minimum holding period from the purchase date to earn at least 10%.

A. 0 years  B. 11 years  C. 12 years  D. 20 years  E. No holding period will earn 10%

Solution.
We know that the length of time we are looking for is the initial duration of the portfolio. Answer C.
Spring 2000 Society of Actuaries Course 6 Examination, Multiple Choice, Problem No. 23

You are given the following information:
• Market value of portfolio X: $10.0 million.
• Modified duration of portfolio X: 6.
• Modified duration of portfolio Y: 4.

Portfolio Y has dollar duration equal to the dollar duration of portfolio X. Calculate the market value of portfolio Y.

A. $6.7 million  B. $10.0 million  C. $15.0 million  D. $40.0 million  E. $60.0 million

Solution.
Recall that modified duration equals dollar duration divided by the market value. Therefore

Dollar duration of portfolio X = Modified duration of portfolio X \cdot Market value of portfolio X = 6 \cdot $10 million =

Dollar duration of portfolio Y = Modified duration of portfolio Y \cdot Market value of portfolio Y = 4 \cdot Market value of portfolio Y.

Therefore

Market value of portfolio Y = \frac{6 \cdot $10 million}{4} = $15 million.

Answer C.
Spring 2000 Society of Actuaries Course 6 Examination, Multiple Choice, Problem No. 35

You are given the following information:

· The following derivatives have the same payment date.
· LIBOR is 9% at the settlement date.

Rank in ascending order (lowest to highest) the value of a single payment of the following derivatives.

I. A floor indexed on LIBOR plus 50 basis points, strike price 11%, notional amount $100,000.
II. A collar indexed on LIBOR plus 100 basis points, strike prices 8% and 11%, notional amount $100,000.
III. A cap indexed on LIBOR plus 50 basis points, strike price 8%, notional amount $125,000.
IV. A corridor indexed on LIBOR plus 0 (zero) basis points, strike prices 8% and 11%, notional amount $200,000.

A. I < IV < III < II B. II < I < III < IV C. II < III < IV < I
D. IV < I < II < III E. IV < II < I < III

Solution.

For the floor described in I, LIBOR plus 50 bps is below its strike level of 11%, and the difference between the two is 1.5%, so the payment is 1.5% \cdot $100,000 = $1,500. For the collar indexed on LIBOR plus 100 basis points, strike prices 8% and 11%, notional amount $100,000, the payment is zero, because LIBOR + 100 bps is at 10%, thus between the two strike rates of 8% and 11%. For a cap indexed on LIBOR plus 50 basis points, strike price 8%, notional amount $125,000, the payment is 1.5% (excess of current rate of 9% + 50 bps over 8%) of $125,000, i.e., $1875. Finally, for a corridor indexed on LIBOR plus 0 (zero) basis points, strike prices 8% and 11%, notional amount $200,000, the payment is the excess of 9% over 8%, i.e., 1% of $200,000 principal, i.e., $2000. The order of magnitude is $0 < $1500 < $1875 < $2000, i.e., II < I < III < IV. Answer B.
Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 14, parts (b) and (c)
You are given the following information about a bond:
• The term to maturity is 2 years.
• The bond has a 9% annual coupon rate, paid semiannually.
• The annual bond-equivalent yield-to-maturity is 8%.
• The par value is $100.
Calculate the (regular, not Macaulay) M-squared of the bond.

A. 1.72  B. 6.78  C. 12.75  D. 20.25  E. 101.75

Solution.
The coupon amount is $4.50. The effective interest rate applicable is 4% per half a year, and if we write \( i \) for the effective annual interest rate, we have \( 1 + i = 1.04^2 \). There are four coupon periods until maturity. Given that, the price of this bond is

\[
4.5a_{\frac{4}{4}\%} + 100 \cdot 1.04^{-4} \approx 101.81.
\]

Macaulay duration of this bond is

\[
\frac{1}{2} \cdot \frac{4.5}{1.04} + \frac{9}{2} \cdot \frac{4.5}{1.04^2} + \frac{3}{2} \cdot \frac{4.5}{1.04^3} + 2 \cdot \frac{104.5}{1.04^4} = 1.87574389.
\]

Regular duration of the bond is

\[
\frac{1}{1.04^2} \cdot \text{Macaulauly duration} = 1.73423067.
\]

Macaulay convexity of this bond is

\[
0.5^2 \cdot \frac{4.5}{1.04} + 1^2 \cdot \frac{4.5}{1.04^2} + 1.5^2 \cdot \frac{4.5}{1.04^3} + 2^2 \cdot \frac{104.5}{1.04^4} = 3.6492821774.
\]

Regular convexity is

\[
C = \frac{1}{(1 + i)^2} D_M + \frac{1}{(1 + i)^2} C_M = \frac{1}{1.04^2} \cdot 1.87574389 + \frac{1}{1.04^2} \cdot 3.6492821774 = 4.722815438.
\]

Therefore, M-squared is

\[
M^2 = C - D^2 = 4.722815438 - 1.73423067^2 = 1.715259421.
\]

Answer A.
Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 15

You are considering the purchase of a 30-year bond with the following characteristics:
• The annual coupon rate is 5.5% payable semiannually.
• The annual bond equivalent yield-to-maturity is 6.25%.
• The annual bond equivalent reinvestment rate is 6.5%.

Your investment horizon is 5 years. The expected annual bond-equivalent yield-to-maturity on 25-year bonds at the end of the investment horizon is 5%. Calculate the total return on an effective annual rate basis for this bond purchase over the 5-year investment horizon.

A. 6.50%  B. 7.60%  C. 8.15%  D. 8.75%  E. 9.10%

Solution.

For simplicity, let us assume that the par value of this bond is $1000. Then the current price of this bond is

\[
\frac{55}{2} \cdot a_{\overline{60}|\frac{6.25}{2}} + 1000 \cdot \left(1 + \frac{6.25}{2}\right)^{-60} \approx 898.94.
\]

Since coupons can be reinvested at 6.5% annual bond equivalent, they will accumulate after five years to

\[
\frac{55}{2} \cdot s_{\overline{10}|\frac{6.50}{2}} \approx 318.91.
\]

Given then prevailing bond-equivalent yield of 5%, the price of the bond at the investment horizon will be

\[
\frac{55}{2} \cdot a_{\overline{50}|\frac{5.0}{2}} + 1000 \cdot \left(1 + \frac{5.0}{2}\right)^{-50} \approx 1070.91.
\]

Total value received by the investor at the end of the investment horizon (5 years) is $318.91 + $1070.91 = $1389.82.

The investor paid $898.94 for this investment, thus the annual effective rate of return is

\[
\left(\frac{1389.82}{898.94}\right)^{\frac{1}{5}} - 1 = 9.105269\%.
\]

Answer E.
Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 16

Assets A, B, and C have the following prices as of January 1, 2000:
- Asset A: $90.70,
- Asset B: $105.69,
- Asset C: $97.53.

The assets have the following risk-free cash flows (given in their entirety):

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$0.00</td>
<td>$100.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>B</td>
<td>$8.00</td>
<td>$108.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>C</td>
<td>$5.00</td>
<td>$5.00</td>
<td>$105.00</td>
</tr>
</tbody>
</table>

Calculate, as of January 1, 2000, the sum of the following nominal annual, compounded semiannually, spot rates:
- from time 0 to time 0.50 years: \( s_{0.50} \),
- from time 0 to time 1 year: \( s_1 \), and
- from time 0 to time 1.5 years: \( s_{1.5} \).

A. 0.22  B. 0.24  C. 0.27  D. 0.29  E. 0.33

Solution.

Based on the information about the bond A:

\[ 90.70 \cdot \left( 1 + \frac{s_1}{2} \right)^2 = 100, \]  
so that

\[ s_1 = 2 \cdot \left( \frac{100}{90.70} \right)^{0.50} - 1 \approx 10.003413\%. \]

Based on the information about the bond B

\[ 105.69 = 8 \cdot \left( 1 + \frac{s_{0.5}}{2} \right)^{-1} + 108 \cdot \left( 1 + \frac{s_1}{2} \right)^{-2}. \]

This gives

\[ s_{0.5} = 2 \cdot \frac{8}{105.69 - 108 \cdot \left( 1 + \frac{s_1}{2} \right)^{-2}} - 1 \approx 6.878717\%. \]

Based on the information about Bond C

\[ 97.53 = 5 \cdot \left( 1 + \frac{s_{0.5}}{2} \right)^{-1} + 5 \cdot \left( 1 + \frac{s_1}{2} \right)^{-2} + 105 \cdot \left( 1 + \frac{s_{1.5}}{2} \right)^{-3}. \]

This gives

\[ s_{1.5} = 2 \cdot \left( \frac{105}{97.53 - 5 \cdot \left( 1 + \frac{s_{0.5}}{2} \right)^{-1} - 5 \cdot \left( 1 + \frac{s_1}{2} \right)^{-2}} \right)^{\frac{1}{3}} - 1 \approx 11.999018\%. \]

The sum of the three is approximately 28.88%.
Answer D.
You are given the following information:

• The current price of Lucent Technologies is $60 per share.
• The two-year forward price is $69.
• For the two-year period, the risk-free rate is a constant annual 5.5%, compounded continuously.
• The stock pays no dividends.

Assume the investor considered can borrow or lend at the risk-free rate. Using only one share of the stock, create an arbitrage strategy. What is the dollar amount of the payoff of that arbitrage the end of two years?

A. $1.88  B. $1.92  C. $1.98  D. $2.02  E. No arbitrage is possible

Solution.

Based on the information about the share price and the risk-free force of interest, the forward price should be

\[ F = S e^{\delta T} = 60 \cdot e^{0.055 \cdot 2} \approx 66.98. \]

This is the price at which the forward transaction would be done if no arbitrage were possible. But the forward price is actually 69, so the forward price is too high. An arbitrage is created by entering into the following transactions:

• Sell a forward contract for a delivery one share of Lucent in two years for $69.
• Borrow $60 at risk-free rate and buy one share of Lucent for $60.

In two years, the share of Lucent purchased for $60 is delivered to satisfy the short forward position. The amount due on the loan is $66.98, while the proceeds from the forward transaction performance are $69. Net to the investor is

\[ 69 - 66.98 = 2.02. \]

Answer D.
Spring 2000 Casualty Actuarial Society Course 8 Examination, Problem No. 24
(some explanations and multiple choice answers added)

You are given the following:

• The risk-free rate is a constant annual 8% compounded continuously.
• The three-month futures price for an asset is $400.
• Dividends on the asset are paid continuously at a constant dividend-yield rate, \( q \), of 2.4% per annum.

What is the current price for the asset, assuming no arbitrage opportunity?

A. $392.08  B. $394.44  C. $389.73  D. $369.25  E. $361.07

Solution.

The basic futures-spot parity formula is \( F = Se^{\delta t} \). But when the underlying produces income, its spot price must be replaced by the current price without that income, e.g., without the dividend. In this case, the stock will yield dividends continuously at the rate of 2.4% per annum, and time to futures maturity is 0.25 years, so that its price of without those dividends is \( Se^{-0.024 \cdot 0.25} \). Given that \( F = $400 \), \( t = 0.25 \text{ years} \), \( \delta = 0.08 \), we obtain

\[
400 = S \cdot e^{-0.024 \cdot 0.25} \cdot e^{0.08 \cdot 0.25}.
\]

Therefore,

\[
S = \frac{400 \cdot e^{0.056 \cdot 0.25}}{e^{0.08 \cdot 0.25}} \approx $394.44.
\]

Answer B.
There is an outstanding call option to buy 100 shares of a company with a strike price of $20. The company then declares a 10% stock dividend. Calculate the number of shares the option holder would have the right to purchase after the stock dividend and the new strike price of the call option after the stock dividend.

A. 100 shares, $20.00  
B. 110 shares, $20.00  
C. 100 shares, $18.18  
D. 110 shares, $18.18  
E. 90 shares, $22.00

Solution.
Since there is no mention of any special arrangements as to the option holder not having the right to the shares resulting from the stock dividend, we can treat the stock dividend as an 11 for 10 stock split. This results in the option contract becoming a contract for 110 shares. But remember that any dividend results in the total value of shares plus dividend equal to the value of shares just a moment before the dividend is declared, so that 100 shares at $20 are worth $2000, and 110 shares must be also worth $2000, with resulting price being approximately $18.18.
Answer D.
Solution.
The put-call parity relationship says that \( C + PV(X) = S + P \). Using the values given in the problem, we get the following left-hand side of the above formula:

\[
5 + 48 \cdot e^{-0.08 \cdot 0.5} \approx 51.12.
\]

The right-hand side of the put-call parity formula equals $50 + $3 = $53, and the two are not equal, even though they must be in the absence of arbitrage. Thus there is an arbitrage opportunity: one must be long the cheap assets and short the expensive ones, i.e., long call plus long \( PV(X) \), and short \( S \) plus short put. If the investor buys the call for $5 and invests $48 \cdot e^{-0.08 \cdot 0.5} \approx $46.12 in a risk-free bond, this will result in a cash outlay of $51.12. By shorting the stock the investor obtains $50 and by writing a put the investor obtains $3, for a total cash inflow of $53. There is a net free cash flow of $1.88, which in six months will accumulate to approximately $1.96. At options expiration, the investor gets $48 from the bond purchased, and uses that and a call held to purchase a share of the stock to cover the short, if the price of a share is above $48 (in this case the put expires worthless and is irrelevant). If the price of a share is below $48, the call expires worthless and is irrelevant, but the investor must use $48 bond proceeds to buy a share for $48 from the party long put, and then can use that share to cover the short. Net cash flow is $1.96. Answer B.
You are given the following information:
• An option market satisfies the condition for put-call parity.
• The current underlying security price is 100.
• A call option with a strike price of 105 and maturity one year from now has a current price of 4.
• A put option with a strike price of 105 and maturity one year from now has a current price of 6.
Determine the short-term risk-free interest rate.

A. 2.9%  
B. 3.9%  
C. 5.9%  
D. 6.9%  
E. 15.4%

Solution.
Put-Call Parity says:
\[ C - P = S - PV(X) \]

or in terms of given data: \( 4 - 6 = 100 - \frac{105}{1+i} \), or \( 102 = \frac{105}{1+i} \), so that \( i = \frac{3}{102} = 2.94\% \).

Answer A.
You are given the following information:

- Projected liability cash flows:
  - Year 1: 43
  - Year 2: 123
  - Year 3: 214
  - Year 4: 25
  - Year 5: 275

- Available assets for investments:
  - 2-year bond with annual coupon of 5%,
  - 3-year bond with annual coupon of 8%,
  - 5-year bond with annual coupon of 10%.

- Face amount of each bond is 100.
- Current market yield curve: 7% for all durations.

Calculate the initial cost to cash flow match the projected liability cash flows utilizing the assets listed above.

A. 525  B. 537  C. 546  D. 558  E. 569

Solution.

There is no need to do any calculation with the assets, as the liabilities must have the same market value as the assets, if they are cash flow matched. All we need to do is calculate the present value of liabilities’ cash flows at 7%. That present value is

\[
\frac{43}{1.07} + \frac{123}{1.07^2} + \frac{214}{1.07^3} + \frac{25}{1.07^4} + \frac{275}{1.07^5} \approx 537.45.
\]

Answer B.
You are given the following information with respect to a callable bond:

<table>
<thead>
<tr>
<th>Time</th>
<th>Expected Cash Flows at a 7% Annual Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.00</td>
</tr>
<tr>
<td>2</td>
<td>7.90</td>
</tr>
<tr>
<td>3</td>
<td>107.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Annual Yield</th>
<th>Bond Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>6%</td>
<td>104.33</td>
</tr>
<tr>
<td>7%</td>
<td>102.37</td>
</tr>
<tr>
<td>8%</td>
<td>99.76</td>
</tr>
</tbody>
</table>

The current yield is 7%. Calculate the ratio of the modified duration (duration with respect to the interest rate) based on expected cash flows to the effective duration estimated based on the data on price changes of the bond available.

A. 1.00  B. 1.07  C. 1.10  D. 1.16  E. 1.25

Solution.
We assume that bond cash flows happen at the end of each year. We then have

Macaulay duration = $\frac{1 \cdot 8 + 2 \cdot 7.90 + 3 \cdot 107.80}{1.07 + 1.07^2 + 1.07^3} \approx 2.7865$.

Modified duration = $\frac{2.7865}{1.07} \approx 2.6042$.

Effective duration estimate

$$D = \frac{A(i - \Delta i) - A(i + \Delta i)}{2A(i) \cdot \Delta i} = \frac{104.33 - 99.76}{2 \cdot 102.37 \cdot 0.01} \approx 2.2321.$$ 

The ratio desired equals

$\frac{2.6042}{2.2321} \approx 1.1667$.

Answer D.
Spring 2001 Casualty Actuarial Society Course 8 Examination, Problem No. 26(a) (multiple choice answers added)

You are given the following information:
• The two-year risk-free interest rate in Canada is 8% per annum (continuously compounded).
• The two-year risk-free interest rate in the U.S. is 5% per annum (continuously compounded).
• The current U.S. Dollar (USD) to Canadian Dollar (CDN) exchange rate is $0.75 USD for $1.00 CDN.

Calculate the two-year forward exchange rate between the Canadian Dollar and the U.S. Dollar so that no arbitrage opportunity exists.

A. 0.71  B. 0.73  C. 0.75  D. 0.77  E. 0.79

Solution.
We treat the Canadian dollar as a stock-like asset producing income at the rate of 8% per annum, continuously compounded. The current price of CDN is $S = 0.75$. Using the forward-spot parity, we have
\[ F = S \cdot e^{(r-d)t} = 0.75 \cdot e^{(0.05-0.08)2} \approx 0.7063234. \]
Answer A.
An insurer is considering the purchase of a portfolio of bonds on January 1, 2001. The annual bond-equivalent yields-to-maturity (i.e., nominal annual rates of return compounded semi-annually), as of January 1, 2001, are as follows:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Yield-to-Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10%</td>
</tr>
<tr>
<td>B</td>
<td>8%</td>
</tr>
<tr>
<td>C</td>
<td>6%</td>
</tr>
</tbody>
</table>

The entire cash flow returns for the bonds are as follows:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Cash Flow at July 1, 2001</th>
<th>Cash Flow at January 1, 2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1,000.00</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>B</td>
<td>$500.00</td>
<td>$500.00</td>
</tr>
<tr>
<td>C</td>
<td>$1,000.00</td>
<td>$1,000.00</td>
</tr>
</tbody>
</table>

Calculate the internal rate of return for this portfolio, expressed as a nominal annual interest rate compounded semi-annually.

A. 7.23%  B. 7.34%  C. 7.45%  D. 7.56%  E. 7.80%

Solution.
The prices of the three bonds considered are

- Bond A: \(\frac{1000}{1.05} \approx 952.38\),
- Bond B: \(\frac{500}{1.04} + \frac{500}{1.04^2} \approx 943.05\),
- Bond C: \(\frac{1000}{1.03^2} \approx 942.60\).

The market value of the total portfolio is the sum of these three, $2,828.03. The yield to maturity, expressed at first as an effective semi-annual rate \(j\), must satisfy the equation

\[
\frac{1500}{1 + j} + \frac{1500}{(1 + j)^2} = 2838.03.
\]

This is a quadratic equation in \(z = (1 + j)^{-1}\), with the solution \(z \approx 0.963564143\), resulting in \(j \approx 0.037813629\). The nominal annual rate of return compounded semiannually is

\[
2j \approx 0.075627258.
\]

Answer D.
You are given the following information about three bonds:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Bond Price</th>
<th>Cash Inflow on:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>on January 1, 2001</td>
<td>July 1, 2001</td>
</tr>
<tr>
<td>A</td>
<td>$96</td>
<td>$100</td>
</tr>
<tr>
<td>B</td>
<td>$88</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>$125</td>
<td></td>
</tr>
</tbody>
</table>

Assuming no arbitrage opportunity exists, calculate the forward rate from July 1, 2002 to January 1, 2003, expressed as an effective annual rate.

A. 8.60%  B. 9.78%  C. 10.82%  D. 11.20%  E. 11.51%

Solution.
Let January 1, 2001 be time 0, and let us count time in half-years at first. We have

\[ 96 \cdot (1 + s_{0,1}) = 100, \]

so that \( s_{0,1} \approx 4.1666667\% \). Furthermore,

\[ 88 \cdot (1 + s_{0,3})^3 = 100, \]

resulting in \( s_{0,3} \approx 4.353201\% \). Finally,

\[ 125 \cdot (1 + s_{0,4})^4 = 150, \]

giving \( s_{0,4} \approx 4.663514\% \). The forward rate from July 1, 2002 to January 1, 2003 must satisfy the equation

\[ (1 + f_{3,4}) \cdot (1 + s_{0,3})^3 = (1 + s_{0,4})^4, \]

so that

\[ f_{3,4} = \frac{(1 + s_{0,4})^4}{(1 + s_{0,3})^3} - 1 \approx 5.60\%. \]

The corresponding effective annual interest rate is

\[ (1 + 0.056)^2 - 1 \approx 11.5136\%. \]

Answer E.
Spring 2001 Casualty Actuarial Society Course 8 Examination, Problem No. 27(b)
(multiple choice answers added)

You are given the following information:
• The two-year zero-coupon interest rate is 7.5%, compounded continuously.
• The three-year zero-coupon interest rate is 8%, compounded continuously.

You enter into a forward rate agreement, where you receive 10% with annual compounding on a principal of $2,000,000 between the end of year two and the end of year three. What is the present value of the forward-rate agreement?

A. $8,675  B. $9,165  C. $9,989  D. $10,995  E. $11,651

Solution.
A forward rate agreement (FRA) is an agreement to pay a specified interest rate at a future time of a specified principal amount. If the rate paid on it equals the current forward rate, its value must be zero, because you can get the same result by shorting (in this case) a two-year bond and going long three-year bond. Thus the value of the excess of interest earned (as we are considering a long-forward position) over the third year in comparison to the current forward rate for that period is the value of the FRA. The current forward rate is determined as
\[ \frac{e^{0.08 \cdot 3}}{e^{0.075 \cdot 2}} - 1 \approx 9.4174\% . \]

The excess interest earned in this agreement is
\[ (10\% - 9.4174\%) \cdot 2,000,000 \approx 11,651.4326. \]

That value is at time 3, and thus the present value is
\[ 11,651.4326 \cdot e^{-0.08 \cdot 3} \approx 9,165.3415. \]

Answer B.
The Chief Financial Officer (CFO) of a large corporation is considering offering an innovative “collared floater” with the following features:

- Value at issue: par.
- Par amount: 10 million.
- Term to maturity: 5 years.
- Coupon: semi-annual payment and reset, 6-month LIBOR + 0.50%.
- Minimum coupon: 7.5%.
- Maximum coupon: 12.5%.

The CFO intends to use derivative instruments to convert this collared floater into synthetic fixed-rate funding. The following quotes for five-year, semi-annual settlement interest rate swaps, caps and floors on 6-month LIBOR are obtained from a market maker in derivative products:

<table>
<thead>
<tr>
<th>Product</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swaps for LIBOR</td>
<td>8.65%</td>
<td>8.75%</td>
</tr>
<tr>
<td>Interest Rate Cap at 12.0%</td>
<td>0.65%</td>
<td>0.75%</td>
</tr>
<tr>
<td>Interest Rate Cap at 12.5%</td>
<td>0.50%</td>
<td>0.60%</td>
</tr>
<tr>
<td>Interest Rate Cap at 13.0%</td>
<td>0.35%</td>
<td>0.45%</td>
</tr>
<tr>
<td>Interest Rate Floor at 7.0%</td>
<td>0.80%</td>
<td>0.90%</td>
</tr>
<tr>
<td>Interest Rate Floor at 7.5%</td>
<td>0.95%</td>
<td>1.05%</td>
</tr>
<tr>
<td>Interest Rate Floor at 8.0%</td>
<td>1.10%</td>
<td>1.20%</td>
</tr>
</tbody>
</table>

Calculate the effective (all-in) interest cost for this synthetic fixed-rate funding. Note: assume the above quotes are annual interest charges on the par amount, and represent the cost as the fixed interest rate obtained plus/minus costs paid for restructuring of the collared floater.

A. 8.75%    B. 9.00%    C. 9.25%    D. 9.50%    E. 9.75%

Solution.

Let $L$ be the value of 6-month LIBOR. The payment made by the corporation is given by:

- $12.5\%$ if $L > 12\%$,
- $L + 0.50\%$ if $7\% \leq L \leq 12\%$,
- $7.5\%$ if $L < 7\%$.

The corporation wants to pay a fixed interest rate, which we will denote by $x\%$, instead. If the total payment is $x\%$, then the payments needed to supplement the existing ones are:

- $x\% - 12.5\% = (x - 0.5)\% - 12\%$ if $L > 12\%$,
- $x\% - (L - 0.50\%) = (x - 0.5)\% - L$ if $7\% \leq L \leq 12\%$,
- $x\% - 7.5\% = (x - 0.5)\% - 7\%$ if $L < 7\%$.

The corporation needs to pay these cash flows. The cash flow of $(x - 0.5)\% - L$ is that of a swap, paying fixed and receiving floating. The ask price for LIBOR swap is 8.75%, so this is what the corporation will have to pay, and based on this $(x - 0.5)\% = 8.75\%$, and $x\% = 9.25\%$. After entering into this swap, paying fixed 8.75%, receiving LIBOR, the corporation will pay...
This means that the corporation, in addition to entering into the swap, must sell a 12% LIBOR cap (as for $L > 12\%$ the corporation’s payment is reduced by $L – 12\%$, and it does not need it, it just wants to pay 9.25% fixed), and buy a 7% LIBOR floor (to cover the extra $7\% – L$ it has to pay for LIBOR below 7%). The swap is costless to enter. The cap brings in income of 65 bps, while the floor costs 90 bps, with the nest cost of the two equal to 25bps. Including this with the fixed rate cost of 9.25%, we obtain the all-in cost of 9.50%.

Answer D.
May 2002 Society of Actuaries Course 6 Examination, Multiple Choice, Problem No. 5
You are given the following information for a 15-year callable bond:
• Annual coupon rate: 9% payable semi-annually.
• Price: 95.32.
• Effective duration: 3.17.
• Half of convexity measure, \( \frac{1}{2} C = \frac{V_t + V_r - 2V_0}{2V_0 \cdot (\Delta y)^2} = -67.31. \)

Calculate the price of the bond after a 50 basis point increase in interest rates.

A. 93.65   B. 93.97   C. 95.32   D. 96.67   E. 96.99

Solution.
Recall the key approximation
\[
\Delta P \approx -D \cdot P \cdot \Delta i + \frac{1}{2} C \cdot P \cdot (\Delta i)^2.
\]

In this case
\[
\Delta P \approx -3.17 \cdot 95.32 \cdot 0.005 + (-67.31) \cdot 95.32 \cdot 0.005^2 \approx -1.67.
\]

Therefore, the new price is 95.32 – 1.67 = 93.65.
Answer A.
You are given the following:
• A stock is currently selling for $20 per share.
• The stock is not expected to pay any dividends over the next 3 years.
• The annual risk-free rate with continuous compounding is 5%.
If the three-year forward price is $23, calculate the present value of a riskless arbitrage profit available.

A. $0.00  B. $0.12  C. $0.20  D. $0.24  E. $3.00

Solution.
The arbitrage-free forward price is $20 \cdot e^{3 \cdot 0.05} \approx 23.24$. Since the price in the market is $23, there is a riskless arbitrage profit of 0.24 available at the time forward is exercised. The present value is $\left( 20 \cdot e^{3 \cdot 0.05} - 23 \right) e^{-3 \cdot 0.05} \approx 20 - 19.80 = 0.20$.

Answer C.
You are given the following with respect to non-callable default-free zero-coupon bonds (with 1,000 par value):

<table>
<thead>
<tr>
<th>Time to Maturity</th>
<th>Current Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>980.392</td>
</tr>
<tr>
<td>2 years</td>
<td>942.596</td>
</tr>
<tr>
<td>3 years</td>
<td>888.996</td>
</tr>
<tr>
<td>4 years</td>
<td>838.561</td>
</tr>
<tr>
<td>5 years</td>
<td>765.134</td>
</tr>
</tbody>
</table>

You are also given the following with respect to a five-year option-free bond: Its annual coupons are $(2 + t)\%$, where $t$ is the time of payment, and are paid at the end of each year, its par value is 1000. For this five-year annual coupon bond, calculate the Macaulay duration and the Macaulay convexity of this bond, at the given yield to maturity.

A. 4.6 and 3.2
B. 3.2 and 22.4
C. 1.2 and 5.2
D. 1.2 and 16.3
E. 4.6 and 22.3

Solution.
This bond pays coupons of: 30, 40, 50, 60, 70, at times 1, 2, 3, 4, 5, respectively, and repays principal of 1000 at time 5. The present values of these payments, calculated using the prices of zero coupon bonds, are:

- Time 1 payment present value: $30 \cdot 0.980392 \approx 29.41176$,
- Time 2 payment present value: $40 \cdot 0.942596 \approx 37.70384$,
- Time 3 payment present value: $50 \cdot 0.888996 \approx 44.4498$,
- Time 4 payment present value: $60 \cdot 0.838561 \approx 50.31366$,
- Time 5 payment present value: $1070 \cdot 0.765134 \approx 818.69338$.

The sum of these present values, i.e., the price of the bond, is approximately 980.57. The Macaulay duration is approximately:

$$D = \frac{1 \cdot 29.41 + 2 \cdot 37.70 + 3 \cdot 44.45 + 4 \cdot 50.31 + 5 \cdot 818.69}{980.57} \approx 4.6227.$$

The Macaulay convexity is approximately:

$$C = \frac{1^2 \cdot 29.41 + 2^2 \cdot 37.70 + 3^2 \cdot 44.45 + 4^2 \cdot 50.31 + 5^2 \cdot 818.69}{980.57} \approx 22.2856.$$

Answer E.
Spring 2003 Society of Actuaries Course 6 Examination, Problem No. C-10
You are given the following with respect to an option-free bond portfolio:
· The value of the bond portfolio using the current yield curve is 800.
· The value of the bond portfolio using the current yield curve with a parallel shift upwards of 20 basis points is 788.
· The value of the bond portfolio using the current yield curve with a parallel shift downwards of 20 basis points is 813.
Using approximated values of duration and convexity, estimate the change in the value of the bond portfolio for a parallel shift upwards of 200 basis points in the yield curve.
A. –9.375% B. –5.725% C. –1.250% D. 0.000% E. 4.500%

Solution.
Based on the information given, we estimate duration of this bond from the formula:
\[ D \approx \frac{P(i - \Delta i) - P(i + \Delta i)}{2P(i)(\Delta i)} = \frac{813 - 788}{2 \times 800 \times 0.002} = \frac{25}{3.2} = 7.8125. \]
The analogous convexity estimate is:
\[ C \approx \frac{P(i - \Delta i) - 2P(i) + P(i + \Delta i)}{P(i) \cdot (\Delta i)^2} = \frac{813 - 2 \times 800 + 788}{800 \times 0.002^2} = 312.5. \]
Based on these two values, and using the approximation formula,
\[ \frac{\Delta P}{P} \approx -D \cdot \Delta i + \frac{1}{2}C \cdot (\Delta i)^2, \]
or equivalently
\[ \Delta P \approx -P \cdot D \cdot \Delta i + \frac{1}{2}P \cdot C \cdot (\Delta i)^2, \]
we get
\[ \Delta P \approx -800 \times 7.8125 \times 0.002 + \frac{1}{2} \times 800 \times 312.5 \times 0.002^2 = -75. \]
This is a loss of 9.375% of the bond value.
Answer A.
A one-year maturity Treasury bond with a semiannual coupon rate of 10% (5% coupon every six months) sells for $102.875. A Treasury Bill with one-year maturity sells for $93.35. Derive the current six-month short rate and the forward rate for the following six-month period.

A. 7.12%, 8.10%  
B. 7.21%, 8.19%  
C. 5.95%, 8.31%  
D. 7.21%, 8.30%  
E. 5.95%, 8.12%

Solution.
Based on the Treasury Bill price, the one-year spot rate is $s_1$ such that

$$93.35 = \frac{100}{1 + s_1},$$

so that $s_1 = 7.123728\%$. If we write $s_{0.5}$ for the six-months spot rate and $f_{0.5,0.5}$ for the forward rate from time 0.5 years to time 1.0 years, then the price of the one-year bond gives us these two equations:

$$102.875 = \frac{5}{(1 + s_{0.5})^{0.5}} + \frac{105}{(1 + s_{0.5})^{0.5}(1 + f_{0.5,0.5})^{0.5}} = \frac{5}{(1 + s_{0.5})^{0.5}} + \frac{105}{1 + s_1}.$$ 

Note that

$$\frac{105}{1 + s_1} = 1.05 \cdot 93.35 = 98.0175.$$ 

Therefore

$$\frac{5}{(1 + s_{0.5})^{0.5}} = 102.875 - 98.0175 = 4.8575,$$

and this results in $s_{0.5} = \left(\frac{5}{4.8575}\right)^2 - 1 = 5.9533\%$. Furthermore,

$$\frac{105}{(1 + s_{0.5})^{0.5}(1 + f_{0.5,0.5})^{0.5}} = 102.875 - 4.8575 = 98.0175,$$

and we proceed to get

$$\frac{21}{(1 + f_{0.5,0.5})^{0.5}} = \frac{98.0175}{4.8575},$$

$$f_{0.5,0.5} = \left(\frac{21 \cdot 4.8575}{98.0175}\right)^2 - 1 \approx 8.3071094\%.$$ 

Answer C.
Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 18(a)

A four-month European call option with a strike price of $60 is selling for $5. The underlying price of stock ABC is currently $61, and the risk-free rate is 12% per annum, compounded continuously. Calculate the value of a four-month European put option with a strike price of $60.

A. $0.95  
B. $1.29  
C. $1.46  
D. $1.65  
E. $1.78

Solution.
We use the put-call parity formula

\[ C - P = S - PV(X) \]  

Using the information given in the problem

\[ 5 - P = 61 - 60e^{-\frac{4}{12}0.12} \]

so that

\[ P = -61 + 60e^{-\frac{4}{12}0.12} + 5 \approx 1.65 \]

Answer D.
Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 30(b) (multiple choice answers added)

A stock index with a continuous dividend yield of 3% per year is currently valued at $800. The futures price for a contract on the index deliverable in six months is $825. The risk-free interest rate is 8% per year with continuous compounding. Find the difference between the arbitrage-free futures price and the actual price.

A. –$4.75  
B. –$2.25  
C. $0.00  
D. $2.25  
E. $4.75

Solution.

The price of the underlying contract without the dividend is

$800 \cdot e^{-\frac{1}{2} \cdot 0.03} \approx 788.99.$

Therefore, the arbitrage-free price of the futures contract is

$788.99 \cdot e^{\frac{1}{2} \cdot 0.08} \approx 820.25.$

The difference sought is $820.25 – 825 = –4.75.$

Answer A.
Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 35 (multiple choice answers and explanation added)

An $80 million interest rate swap has a remaining life of 15 months. Under the terms of the swap, six-month LIBOR is exchanged for 10% per annum (compounded semiannually). LIBOR rates with continuous compounding have been the same for the past 3 months and are shown by maturity below.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Annual LIBOR rate (continuous compounding)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 months</td>
<td>8.00%</td>
</tr>
<tr>
<td>6 months</td>
<td>8.25%</td>
</tr>
<tr>
<td>9 months</td>
<td>8.50%</td>
</tr>
<tr>
<td>12 months</td>
<td>8.75%</td>
</tr>
<tr>
<td>15 months</td>
<td>9.00%</td>
</tr>
</tbody>
</table>

Calculate the current value of the swap to the party paying floating rates (i.e., the party short the swap). Assume that the floating payment at the applicable rate is made at the end of the period for which the floating rate is calculated, and that both sides’ payments are made every six months.

A. – $1.52 million  B. – $1.02 million  C. – $0.01 million  D. $1.02 million  E. $1.49 million

Solution.

There are 15 months remaining in the swap, thus the last payment exchange was 3 months ago, and the next one will be in three months. The next floating payment is the interest due on $80 million, at 6-month LIBOR from three months ago, identical to the current LIBOR of 8.25% compounded continuously. Therefore, that amount of interest is

$$e^{0.0825 \cdot 0.5} - 1) \cdot \$80,000,000 \approx \$3,369,000.09.$$

This will be paid in three months. Additionally, in three months, the floating payments will refer to $80 million principal, at then market rate of LIBOR, so the principal amount of $80 million will be worth $80 million in terms of market value. Therefore, the current value of the floating side payments is

$$\left(\$3,369,008.09 + \$80,000,000\right) \cdot e^{-0.08 \cdot 0.25} \approx \$81,718,191.10.$$

The remaining fixed side payments are 0.05 \cdot \$80,000,000 = \$4,000,000 in three months, nine months and fifteen months, plus the repayment of principal of $80,000,000 in 15 months, and have the present value of

$$\$4,000,000 \cdot e^{-0.08 \cdot 0.25} + \$4,000,000 \cdot e^{-0.08 \cdot 0.75} + \$84,000,000 \cdot e^{-0.09 \cdot 1.25} \approx \$82,735,930.00.$$

The value of the swap to the party paying floating and receiving fixed (i.e., short a short-term bond and long a long-term bond) is approximately

$$\$82,735,930.00 - \$81,718,191.10 = \$1,017,738.85.$$

Answer D.
Suppose that a portfolio $P$ is worth $80$ million and the S&P 500 Index is at 1000. The value of the portfolio mirrors the value of the index, and each option contract is for $100$ times the index. What type of options and how many of these options should be purchased to provide protection against the value of the portfolio falling below $72$ million in one year?

A. 800 puts with exercise price of 900  
B. 900 puts with exercise price of 800  
C. 1000 puts with exercise price of 800  
D. 900 calls with exercise price of 1000  
E. 800 calls with exercise price of 900  

Solution. 
If the portfolio falls from $80$ million to $72$ million, this represents a 10% decline in value. We want an option contract, which will protect the entire portfolio from dropping more than 10% than its current value. This will be achieved by a portfolio of puts with the same total exercise price, for all contracts combined, as the value of the total portfolio after a 10% decline. Each option corresponds to $100,000$ in the current value of the portfolio, so for $80$ million we need 800 option contracts. The contracts must be for a 10% decline from the current 1000 value of the index, i.e., for exercise price of 900. Answer A.
Spring 2003 Casualty Actuarial Society Course 8 Examination, Problem No. 37(a)  
(multiple choice answers added)

Suppose that a portfolio $P$ is worth $80$ million and the S&P 500 Index is at 1000. The value of the portfolio mirrors the value of the index, and each option contract is for $100$ times the index. What type of options and how many of these options should be purchased to provide protection against the value of the portfolio falling below $72$ million in one year?

A. 800 puts with exercise price of 900  
B. 900 puts with exercise price of 800  
C. 1000 puts with exercise price of 800  
D. 900 calls with exercise price of 1000  
E. 800 calls with exercise price of 900

Solution.  
If the portfolio falls from $80$ million to $72$ million, this represents a 10% decline in value. We want an option contract, which will protect the entire portfolio from dropping more than 10% than its current value. This will be achieved by a portfolio of puts with the same total exercise price, for all contracts combined, as the value of the total portfolio after a 10% decline. Each option corresponds to $100,000 in the current value of the portfolio, so for $80$ million we need 800 option contracts. The contracts must be for a 10% decline from the current 1000 value of the index, i.e., for exercise price of 900. Answer A.
You are given the following with respect to T-Bills issued on February 1, 2004:

<table>
<thead>
<tr>
<th>Term to Maturity</th>
<th>Spot Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 month</td>
<td>98.50</td>
</tr>
<tr>
<td>2 months</td>
<td>98.40</td>
</tr>
<tr>
<td>3 months</td>
<td>98.20</td>
</tr>
<tr>
<td>4 months</td>
<td>98.00</td>
</tr>
</tbody>
</table>

A futures contract is available for delivery of a 3-month T-Bill on March 1, 2004. Determine the implied price of the futures contract.


Solution.
The current 4-month T-Bill will become a 3-month T-Bill on March 1, 2004. Therefore the current price of the underlying is 98. The implied futures price is the spot price accumulated at the risk-free rate one month into the future, and that accumulation can be achieved by dividing by the current price of a 1-month T-Bill. Therefore, the answer is:

\[
98 \cdot \frac{100}{98.50} \approx 99.49.
\]

Answer D.
The current price of copper is $0.85 per pound. The storage costs are $0.08 per pound per year, payable quarterly in advance. The current estimated convenience yield is 10.0% of the spot price. Assume that interest rates are 5% per annum, with continuous compounding, for all maturities. Calculate the futures price of copper for delivery in nine months.

A. $0.91  B. $0.90  C. $0.89  D. $0.88  E. $0.87

Solution.
The storage costs should be considered to be a negative dividend. The total storage costs have the present value of

\[ 0.02 + 0.02e^{-0.25 \cdot 0.05} + 0.02e^{-0.50 \cdot 0.05} = 0.059. \]

The “ex-dividend” spot price is $0.85 + $0.059 = $0.909. The convenience yield should be treated like a positive dividend. Thus the “ex-dividend” spot price \( S \), after consideration of both the storage costs and the convenience yield is

\[ S = 0.909 \cdot e^{-0.10 \cdot \frac{9}{12}} \approx 0.843318829. \]

Using the futures-spot parity formula, we obtain the futures price to be

\[ F = Se^{-r} \approx 0.843318829 \cdot e^{0.05 \cdot \frac{9}{12}} \approx 0.875543726 \approx $0.88. \]

Answer D.
Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 27
(multiple choice answers added)
A stock price is currently $50.00. It is known that at the end of two months, it will be either $54.00 or $46.00. The risk-free interest rate is 9.0% per annum with continuous compounding. Calculate the value of a two-month European call option with a strike price of $48 on this stock.

A. $2.08  B. $2.89  C. $3.26  D. $3.51  E. $3.99

Solution.
The risk neutral probability of the up move in the stock price is
\[ p = \frac{e^{0.09 \frac{2}{12}} \cdot \frac{46}{50} - \frac{46}{50}}{\frac{54}{50} - \frac{50}{50}} \approx 0.594456654. \]
In the case of the up move, the call will pay $6, and in the case of the down move, it will pay nothing. Therefore, the call’s price is
\[ p \cdot$6 \cdot e^{-0.09 \frac{2}{12}} \approx 3.51363808. \]
Answer D.
A bond has a Macaulay duration of 10 and a convexity of 400. Using annual compounding, the yield-to-maturity for this bond is 8.0%. Using modified duration and convexity, estimate the percentage change of this bond for a 75 basis point rise in yield-to-maturity.

A. −9.26%  B. −7.50%  C. −6.94%  D. −5.82%  E. −4.62%

Solution.

The modified duration of this bond equals \( \frac{10}{1.08} \). The approximate change in the value of the bond is given by the formula

\[
\frac{\Delta P}{P} \approx -D \cdot \Delta i + \frac{1}{2} \cdot C \cdot (\Delta i)^2.
\]

In this case, we get

\[
\frac{\Delta P}{P} \approx -10 \cdot 0.0075 + \frac{1}{2} \cdot 400 \cdot (0.0075)^2 \approx -5.819444\%.
\]

Answer D.
A stock is expected to pay a dividend of $3 per share in two months and again in eight months. The stock price is $300 and annual risk-free rate of interest is 5.0% with continuous compounding for all maturities. An investor has just taken a long position in a nine-month forward contract on the stock. Calculate the forward price.

A. $301.50  
B. $303.07  
C. $304.25  
D. $305.36  
E. $306.00

Solution.
The present value of dividends to be paid by the stock during the forward contract term is

$3 \cdot e^{-0.05 \cdot \frac{2}{12}} + 3 \cdot e^{-0.05 \cdot \frac{8}{12}} \approx 5.87675218.$

The current ex-dividend stock price is $300 – $5.87675218, so that the forward price is

$(300 – 5.87675218) \cdot e^{0.05 \cdot \frac{9}{12}} \approx 305.362285.$

Answer D.
Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 28
(multiple choice answers added)

Consider a stock with a current price of $45.00. At the end of six months, it can either go up 6.0% or down 6.0%. You have sold 1000 six-month European call options on this stock with a strike price of $46.00. Calculate the number of shares of stock that you would need to purchase in order to create a risk-less hedge to replicate the short position in the 1000 six-month European call options.

A. 315  B. 320  C. 325  D. 330  E. 335

Solution.
The number of shares of stock per option is
\[ \beta = \frac{f_u - f_d}{u - d} \cdot \frac{1}{S} = \frac{(45 \cdot 1.06 - 46) - 0}{1.06 - 0.94} \cdot \frac{1}{45} = 0.314814815. \]
Since we are considering 1000 options, we need approximately 314.8148 shares.
Answer A.
Spring 2004 Casualty Actuarial Society Exam 8, Problem No. 32
A bond has Macaulay duration of 10.0 and a convexity of 400. Using annual compounding, the yield to maturity for this bond is 8.0%. Using modified duration and convexity, estimate the percentage price change of this bond for a 75 basis point rise in yield-to-maturity.
A. – 5.8%  B. – 4.6%  C. 0.2%  D. 3.3%  E. 4.5%

Solution.
The (modified) duration of this bond is
\[ D = \frac{D_M}{1 + i} = \frac{10}{1.08} = 9.259. \]
Since the convexity measure given is the regular one, not the Macaulay convexity, we can estimate the price change using it, and get
\[ \frac{\Delta P}{P} \approx -D \cdot \Delta i + \frac{1}{2} \cdot C \cdot (\Delta i)^2 = (-9.259) \cdot 0.0075 + 0.50 \cdot 400 \cdot 0.0075^2 = -0.05819. \]
This means that as rates rise by 75 basic points, the price of this bond is estimated to fall by 5.819%.
Answer A.
Spring 2004 Casualty Actuarial Society Course 8 Examination, Problem No. 19a
(multiple choice answers added)
The three-month interest rate in Japan is 2.0% per annum and in the United States it is 5.0% per annum, both with continuous compounding. The spot price of the Japanese Yen is $0.0085. Calculate the theoretical futures price for this contract.

A. $0.008607  B. $0.00856  C. $0.00865  D. $0.00854  E. $0.00849

Solution.
Based on futures-spot parity the theoretical futures price is

\[ 0.0085 \cdot e^{(0.05 - 0.02) \frac{3}{12}} = 0.00856. \]

Answer B.
Spring 2004 Casualty Actuarial Society Exam 8, Problem No. 33

The loss and allocated loss adjustment expense payout for a property-casualty insurance company is as follows:

<table>
<thead>
<tr>
<th>Payout year</th>
<th>Cumulative Percent Paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
</tr>
<tr>
<td>3</td>
<td>50%</td>
</tr>
<tr>
<td>4</td>
<td>75%</td>
</tr>
<tr>
<td>5</td>
<td>100%</td>
</tr>
</tbody>
</table>

We assume that all loss and allocated expense payments are made in mid-year. Equivalently, this means that if there is a $100 loss to be paid, $10 of it is paid at time 0.5, $10 at time 1.5, $30 at time 2.5, $25 at time 3.5, and $25 at time 4.5. Calculate the Macaulay duration of liabilities using a 5.0% interest rate.

A. 0.80  B. 1.80  C. 2.80  D. 3.80  E. 4.80

Solution.

We can basically assume that the cash flows are as identified in the problem: $10 of it is paid at time 0.5, $10 at time 1.5, $30 at time 2.5, $25 at time 3.5, and $25 at time 4.5. The Macaulay duration is then given by its basic formula:

\[
D_M = \frac{\sum_{t>0} t \cdot PV(CF_t)}{\sum_{t>0} PV(CF_t)} =
\]

\[
= \frac{0.50 \cdot \frac{10}{1.05^{0.50}} + 1.5 \cdot \frac{10}{1.05^{1.50}} + 2.5 \cdot \frac{30}{1.05^{2.50}} + 3.5 \cdot \frac{25}{1.05^{3.50}} + 4.5 \cdot \frac{25}{1.05^{4.50}}}{\frac{10}{1.05^{0.50}} + \frac{10}{1.05^{1.50}} + \frac{30}{1.05^{2.50}} + \frac{25}{1.05^{3.50}} + \frac{25}{1.05^{4.50}}} =
\]

\[
\approx 2.87355.
\]

Answer C.
Spring 2004 Society of Actuaries Course 6 examination, Problem No. B-32
You are given the following information with respect to a portfolio:
• Portfolio Macaulay duration is 3.
• Portfolio Macaulay dispersion \( (M^2) \) is 0.75.
• All cash flows are positive.
There are only cash flows at times 1, 2, 3, 4, and 5, whose present values are, respectively, 1, \( X \), \( Y \), 0, and 1. Given this information, determine the value of \( Y \).

A. 0  
B. 3  
C. 4  
D. 9  
E. 26

Solution.
The total present value of this portfolio is \( X + Y + 2 \). Its Macaulay duration is
\[
3 = \frac{1 \cdot 1 + 2 \cdot X + 3 \cdot Y + 5 \cdot 1}{X + Y + 2}.
\]
This implies that
\[
3X + 3Y + 6 = 6 + 2X + 3Y.
\]
Therefore, \( X = 0 \). The Macaulay convexity is:
\[
C_M = \frac{1^2 \cdot 1 + 3^2 \cdot Y + 5^2 \cdot 1}{Y + 2} = \frac{26 + 9Y}{Y + 2}.
\]
The Macaulay dispersion is:
\[
0.75 = M^2_M = C_M - D^2_M = \frac{26 + 9Y}{Y + 2} - 9.
\]
Therefore,
\[
9.75 = \frac{26 + 9Y}{Y + 2}.
\]
We solve this equation to get \( Y = \frac{8}{3} \).
Answer D.
You are given the following information:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Term</th>
<th>Effective Duration</th>
<th>Effective Convexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>3.1</td>
<td>–41.7</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>4.5</td>
<td>23.4</td>
</tr>
</tbody>
</table>

Calculate the approximate percentage price change for Bonds A and B assuming a decrease in yield of 0.50%.

A. Bond A: 1.50%, Bond B: 1.60%
B. Bond A: 1.50%, Bond B: 2.22%
C. Bond A: 2.22%, Bond B: 1.50%
D. Bond A: 2.22%, Bond B: 2.28%
E. Bond A: 1.50%, Bond B: 2.28%

Solution.
The key formula is:

\[
\frac{\Delta P}{P} \approx -D \cdot \Delta i + \frac{1}{2} \cdot C \cdot (\Delta i)^2.
\]

For Bond A

\[
\frac{\Delta P}{P} \approx -3.1 \cdot (-0.005) + \frac{1}{2} \cdot (-41.7) \cdot (0.005)^2 \approx 0.01497875 \approx 1.50%.
\]

For Bond B

\[
\frac{\Delta P}{P} \approx -4.5 \cdot (-0.005) + \frac{1}{2} \cdot 23.4 \cdot (0.005)^2 \approx 0.02279375 \approx 2.28%.
\]

As expected, for a bond with negative convexity appreciates less when rates decline.

Answer E.
Spring 2005 Society of Actuaries Course 6 Examination, Problem No. 4, Part c
You are given the following with respect to Treasury securities as of today, May 13, 2005:

<table>
<thead>
<tr>
<th>Security</th>
<th>Years to Maturity</th>
<th>Annual Coupon Rate</th>
<th>Yield-to-maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>0%</td>
<td>3.2%</td>
</tr>
<tr>
<td>C</td>
<td>1.5</td>
<td>6%</td>
<td>3.5%</td>
</tr>
<tr>
<td>D</td>
<td>2.0</td>
<td>5%</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Calculate the one-year forward rate, one year from today.

A. 3.52%  B. 3.65%  C. 3.76%  D. 3.91%  E. 4.02%

Solution.
We see that the spot rate from time 0 to time 0.5 is 3.0% per annum, and from time 0 to time 1 is 3.2% per annum. Per $100 par, security C pays coupons of $3 at times 0.5, 1, and 1.5, as well as its principal of $100 at time 1.5. Given that C’s yield to maturity is 3.5%, if we write \( s_{1.5} \) for the spot rate from time 0 to time 1.5, we get

\[
\frac{3}{1.03^{0.5}} + \frac{3}{1.032} + \frac{103}{(1 + s_{1.5})^{1.5}} = \frac{3}{1.03^{0.5}} + \frac{3}{1.035} + \frac{103}{1.035^{1.5}} \approx 103.67.
\]

This gives us \( \frac{103}{(1 + s_{1.5})^{1.5}} \approx 97.83 \). Therefore, \( s_{1.5} \approx 3.49103\% \). Now we do the same analysis for the spot rate from time 0 to time 2, denoted by \( s_{2} \), and security D:

\[
\frac{2.5}{1.03^{0.5}} + \frac{2.5}{1.032} + \frac{2.5}{1.0349103^{1.5}} + \frac{102.5}{(1 + s_{2})^2} \approx 7.26 + \frac{102.5}{(1 + s_{2})^2} =
\]

\[
= \frac{2.5}{1.036^{0.5}} + \frac{2.5}{1.036} + \frac{2.5}{1.036^{1.5}} + \frac{102.5}{1.036^2} \approx 102.74.
\]

Therefore

\[
s_{2} = \sqrt{\frac{102.5}{102.74 - 7.26}} - 1 \approx 3.610981\%.
\]

Since the spot rate from time 0 to time 1 is \( s_{1} = 3.2\% \), the forward rate \( f_{1,2} \) from time 1 to time 2 is equal to

\[
f_{1,2} = \frac{(1 + s_{2})^2}{1 + s_{1}} - 1 = \frac{(1.03610981)^2}{1.032} - 1 \approx 4.023599\%.
\]

Answer E.
A fixed-rate bond with a market value of 20 million and duration of 4 is separated into three bonds. Two of the bonds are floaters and the third is an inverse floater. You are given the following information with respect to the floaters:

<table>
<thead>
<tr>
<th>Floater</th>
<th>Market Value</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16 million</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>2 million</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Calculate the duration of the inverse floater.

A. 2.50  B. 3.06  C. 3.15  D. 25.20  E. 31.50

Solution.
The key insight is that the duration of a portfolio is a market-value-weighted average of durations of individual pieces in the portfolio. In this case, the fixed rate bond is the portfolio, composed of three pieces, two floaters and one inverse-floater. Therefore, if we denote by $D$ the duration of the inverse floater, we obtain the following equation

\[
4 = \frac{16}{20} \cdot 1 + \frac{2}{20} \cdot 0.5 + \frac{20 - 16 - 2}{20} \cdot D.
\]

This gives $4 = 0.8 + 0.05 + 0.1 \cdot D$, and from this we calculate $D = 31.5$. Answer E.
Spring 2005 Society of Actuaries Course 6 Examination, Problem No. 12
You are given the following with respect to Stock X:
• Stock price today: 10.
• Stock price one year from today: either 12 or 7.
• Call option strike price: 11.
The annual interest rate is 5%. Calculate the no-arbitrage call option price on Stock X as of today.
A. 0.67        B. 0.74        C. 1.40        D. 1.47        E. 3.33

Solution.
The risk-neutral probability of the up move in the stock price is
\[ p = \frac{(1 + i) - d}{u - d} = \frac{1.05 - \frac{7}{10}}{\frac{12}{10} - \frac{7}{10}} = \frac{0.35}{0.50} = 0.70. \]
The call option pays 1 in the up state and 0 in the down state. Therefore, its arbitrage-free price is \( 0.7 \cdot \frac{1}{1.05} \approx 0.6667. \)
Answer A.
You are given the following:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Market Value</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>100</td>
<td>5.2</td>
</tr>
<tr>
<td>Liabilities</td>
<td>85</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Estimate the change in economic surplus if interest rates decline by 50 basis points.

A. –1.5  B. –0.7  C. 0.0  D. 0.7  E. 1.5

Solution.
The surplus before the change in interest rates is \( S = 100 - 85 = 15 \). Based on the durations of assets and liabilities we estimate

\[
\frac{\Delta A}{A} \approx -D(A) \cdot \Delta i = -5.2 \cdot (-0.005) = 0.026 = 2.6\% ,
\]

\[
\frac{\Delta L}{L} \approx -D(L) \cdot \Delta i = -4.4 \cdot (-0.005) = 0.022 = 2.2\% .
\]

Therefore, the new values of assets is approximated as \( 100 \cdot 1.026 = 102.60 \), and the new value of liabilities is approximated as \( 85 \cdot 1.022 = 86.87 \). The new value of surplus is \( 102.60 - 86.87 = 15.73 \). This gives the change in economic surplus as \( 15.73 - 15.00 = 0.73 \). Alternatively, you can just directly calculate the duration of the surplus as

\[
\frac{100}{15} \cdot 5.2 \frac{85}{15} \cdot 4.4 \approx 9.73333333 .
\]

Therefore,

\[
\frac{\Delta S}{S} \approx -D(S) \cdot \Delta i = -9.73333333 \cdot (-0.005) \approx 4.8666667\% ,
\]

and this 4.866667% increase in surplus is \( 4.866667\% \cdot 15 = 0.73 \).

Answer D.
Spring 2005 Casualty Actuarial Society Course 8 Examination, Problem No. 20

At the beginning of the year, an investor buys a zero-coupon bond for $400 with a par value of $1,000 and 10 years until maturity. This investor’s tax rate on interest income is 35% and on capital gains is 20%. During the year, while the investor holds the bond, the yield to maturity changes to 8.0%. Determine the investor’s after-tax return on the bond if sold at the end of the year. Additionally, calculate how much tax the investor owes at the end of the year if the bond is not sold.

A. 18.61%, $13.43
B. 9.60%, $13.43
C. 12.89%, $12.50
D. 9.60%, $12.50
E. 18.61%, $11.79

Solution.

Consider all yields to be annual. The annual yield to maturity as of the date of purchase is

\[
\left( \frac{1000}{400} \right)^{\frac{1}{10}} - 1 \approx 9.5958\%.
\]

This is the rate, which is used for calculation of imputed interest income, as prescribed by the Internal Revenue Code. The expected value of the bond at the end of the first year is

\[
400 \cdot 1.095958 \approx 438.38,
\]

and this gives $38.38 of imputed interest income. The actual value of the bond is

\[
1000 \cdot 1.08^{-9} \approx 500.25.
\]

Thus the investor enjoys $500.25 – $438.38 = $61.87 of capital gain. After-tax income is

\[
38.38 \cdot (1 - 0.35) + 61.87 \cdot (1 - 0.2) = 74.44.
\]

After-tax rate of return is therefore

\[
\frac{74.44}{400} \approx 18.61\%.
\]

If the bond is not sold, the investor only pays tax on the imputed interest and the amount of taxes is

\[
38.38 \cdot 0.35 = 13.43.
\]

Answer A.
Consider a one-year long forward contract on a bond. This bond matures in 5 years and has a 7% coupon rate, with coupon paid semi-annually. The current price of the bond is $850, and its par value is $1,000. Assume that the continuously compounded risk-free interest rate is 10% per annum. Calculate the arbitrage-free price of the forward contract.

A. $939.40  B. $935.00  C. $875.50  D. $867.60  E. $825.24

Solution.
The standard forward-spot parity relationship is $F = S e^{rt}$, where $F$ is the forward price, $S$ is the spot price, $r$ is the risk-free force of interest, and $t$ is the time to contract expiration. However, the underlying produces income during the period of the forward contract, so the spot price must be without that income. The present value of the income produced by the underlying is:

$$35e^{-0.5 \cdot 0.1} + 35e^{-1 \cdot 0.1} \approx 64.96.$$ 

Therefore, the price of the underlying without income is

$$850 - 64.96 = 785.04.$$ 

The forward price is therefore

$$F = 785.04 e^{0.1} \approx 867.60.$$ 

Answer D.
An investor contacts her broker to enter into five long futures contracts on bushels of corn. Each contract is for the delivery of 5,000 bushels. The current futures price is $2.40 per bushel. The initial margin is $2,000 per contract. The maintenance margin is $1,500 per contract. Calculate the futures price of corn at which the investor could withdraw $2,000 from the margin account.

A. $2.38  B. $2.40  C. $2.42  D. $2.44  E. $2.46

Solution.
Five contracts for 5,000 bushels total 25,000 bushels. The current futures price is $2.40 per bushel, for a total futures price for 25,000 bushels for the entire position of

\[ 25,000 \times 2.40 = 60,000. \]

The investor posts initial margin of five times $2,000, i.e., $10,000. Following that, maintenance margin is five times $1,500, i.e., $7,500. This means that once the trade is placed, $2,500 will become available immediately. But for any money to be available, the investor will have to wait till the end of the first day of trading. The investor will be able to withdraw $2,000 exactly if a loss of $500 exactly is produced at the end of the day. For that loss to be produced, futures price must decrease by

\[ \frac{500}{25,000} = 0.02. \]

Thus the futures price must be $2.38. Note that the solution published by the CAS assumes that the $2,000 withdrawal must be from a gain on the trade, but the problem does not say anywhere that the withdrawal must be produced by a gain.

Answer A.
A bank offers a corporate client a choice between borrowing cash at 10% per annum and borrowing platinum at 2.5% per annum. If platinum is borrowed, interest must be repaid in platinum. Thus 100 ounces borrowed today would require 102.5 ounces to be repaid in one year. The risk-free interest rate is 7% per annum, and storage costs for platinum are 0.4% per annum, continuously compounded. Assume the following:

- The interest rates on the loans are expressed with annual compounding.
- The risk-free interest rate and storage costs are expressed with continuous compounding.
- There is no income earned on platinum.
- There are no transaction costs for trading.
- The market participants can borrow money at the same risk-free rate of interest at which they can lend money.

Determine whether the rate of interest on the platinum loan is too high or too low in relation to the rate of interest on the cash loan, and then calculate the difference between the current rate (2.5% annually compounded) and the rate that would make the two loans equivalent.

A. 0.37%  B. 0.13%  C. 0.00%  D. –0.13%  E. –0.37%

Solution.
If we replicate the pure platinum transaction (borrow 1 ounce of platinum, repay 1.025 ounces of platinum) with market transaction involving money and platinum (buy 1 ounce of platinum with borrowed money now, sell 1.025 ounces of platinum in a year, repay the cash loan), the net result will give us the answer we are seeking. Note that the original platinum transaction and its replication involve paying the same storage cost for platinum, so that the direct storage cost will not matter the calculation (but its indirect effect in forward pricing may). Recall that storage cost is treated as a negative dividend in forward pricing. Let us write $S_0$ for the current spot price of one ounce of platinum. The platinum loan has the net effect of the corporate client spending $S_0$ to buy an ounce of platinum today, and then committing to sell 1.025 ounces in a year on the forward. We know that the current forward price is $S_0 e^{0.07+0.004}$. The effective annual return on money on this transaction (earned by the bank, from which platinum is borrowed, and paid by the corporate client, if borrowing in platinum) is

$$\frac{1.025S_0 \cdot e^{0.07+0.004}}{S_0} - 1 = 10.372698\%.$$ 

But the corporate client pays 10% to borrow money, thus we see that the platinum loan results in money cost of approximately 10.372698%. The platinum loan rate is too high. By borrowing money at 10%, buying an ounce of platinum now and selling 1.025 ounces of platinum on the forward, the corporate client could earn, per ounce, approximately $1.025S_0 \cdot e^{0.07+0.004} - 1.1S_0 \approx 0.00372698S_0$, or, as a fraction of the price of an ounce of platinum 0.372698%. We conclude that the platinum loan rate is approximately 37 basis points too high.

Answer A.
ABC Insurance Company has entered into a 10-year swap with XYZ Insurance Company. Under the terms of the swap, ABC receives interest at 4% per annum in Swiss francs and pays interest at 8% per annum in U.S. dollars. Interest payments are exchanged once per year. The principal amounts are 10 million dollars and 13 million francs. Suppose that XYZ declares bankruptcy at the end of year 7, right before the swap payment is to be made. The exchange rate at the end of year 7 is $0.80 per franc. Assume that forward rates are realized and, at the end of year 7, the interest rate is 4% per annum in Swiss francs and 9% per annum in U.S. dollars for all maturities. All interest rates are quoted with annual compounding. Calculate the cost to ABC at the end of year 7 due to XYZ’s bankruptcy.

A. $0       B. $270,000       C. $816,000       D. $1,570,000       E. $10,816,000

Solution.
The cost to ABC is the difference between what it is owed in remaining payments by XYZ, and what it owes to XYZ. At the end of year 7, ABC is owed the following amount in Swiss francs (using the interest rates prevailing then and cash flows promised):

\[ 0.04 \cdot 13,000,000 \cdot \overline{a}_{4\%} + 13,000,000 \cdot 1.04^{-3}. \]

The exchange rate is then 0.80 francs per dollar, so that the amount owed ABC in U.S. dollars is

\[ 0.80 \cdot \left( 0.04 \cdot 13,000,000 \cdot \overline{a}_{4\%} + 13,000,000 \cdot 1.04^{-3} \right) = 10,816,000. \]

The amount ABC owes XYZ, in U.S. dollars, is

\[ 0.08 \cdot 10,000,000 \cdot \overline{a}_{9\%} + 10,000,000 \cdot 1.09^{-3} = 10,546,870.53. \]

The net amount owed ABC is

\[ $10,816,000 - $10,546,870.53 = $269,129.47. \]

Answer B.
The yield-to-maturity on one-year zero-coupon bond is currently 7%. The yield-to-maturity on two-year zero-coupon bond is currently 8%. The Treasury plans to issue a two-year maturity coupon bond, paying coupons once per year with a coupon rate of 9%. The face value of the bond is $100. Calculate the two-year coupon bond’s yield to maturity.

A. 7.99%  B. 7.96%  C. 7.93%  D. 7.91%  E. 7.88%

Solution.
The price of the two-year coupon bond, based on the spot interest rates, is equal to

\[
\frac{9}{1.07} + \frac{109}{1.08^2} \approx 101.86.
\]

Its yield-to-maturity \( i \) (annual effective rate) must satisfy the equation

\[
\frac{9}{1 + i} + \frac{109}{(1 + i)^2} = 101.86.
\]

This gives \( i \approx 7.956942\% \).

Answer B.
You are given the following information for a bond portfolio:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Term (Years)</th>
<th>Annual Coupon Rate (Paid Semi-Annually)</th>
<th>Price</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>0.0%</td>
<td>122</td>
<td>200</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>4.8%</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>Floating equal to one-year Treasury Bill rate plus 50 bps reset annually</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

The floating rate has just been reset based on the one-year Treasury Bill rate of 4% (bond equivalent yield). Using effective duration of this portfolio, given as 4.78204163, estimate the price of the portfolio given a 50 basis point parallel upward shift in the yield curve.

A. 329.70   B. 326.60   C. 322   D. 317.40   E. 314.30

Solution.
The formula telling us what that change is

\[ P(i + \Delta i) \approx P(i) - D \cdot P(i) \cdot \Delta i. \]

We are given \( P(i) = 322, \ D = 4.78204163, \) and \( \Delta i = 0.005. \) Therefore

\[ P(i + \Delta i) \approx 322 - 4.78204163 \cdot 322 \cdot 0.005 \approx 314.30. \]

Answer E.
A firm is short a LIBOR participating cap at a strike rate of 8% with a participation rate of 80%. LIBOR rises to 10%. Determine the firm’s payment to the counterparty.

A. 0% of LIBOR  B. 4% of LIBOR  C. 16% of LIBOR  D. 20% of LIBOR  E. 25% of LIBOR

Solution.
The excess of LIBOR over 8% is 2%. Participation rate applies to the case when the rate is below the cap rate, and the only payment here is the excess of 10% over 8%, i.e., 2%, which is 25% of LIBOR.
Answer E.

Explanation:
Similar to a regular cap, a participating cap provides protection from floating rates rising above a specified maximum cap level, but a participating cap requires no upfront fee, and instead the buyer of a cap agrees to forgo a portion of the rate benefit when floating rates decline. For example, in a 50% participating cap at 10%, if floating rate is about 10%, the holder of a cap is paid the excess over 10%, and if the floating rate is below 10%, the holder of a cap has to actually pay 50% of the excess of 10% over the floating rate. A participating floor is structured similarly.
You are given the following with respect to a stock:
- The one-year call option has a strike price of 25.
- The value of the put option is 5.
- The continuously compounded risk-free rate of return is 10%.

Using put-call parity, calculate the difference between the price of the stock and price of the one-year call option.

A. 17.62  B. 20.00  C. 22.63  D. 27.62  E. 32.63

Solution.
Let $C$ denote the price of the call option, $X = 25$ be the strike price, $S$ be the current price of the underlying stock, $t = 1$ be the time to expiration of options considered, $P = 5$ be the price of the put option, and $\delta = 10\%$ be the risk-free force of interest. The put-call parity formula is

$$C - P = S - PV(X).$$

We are interested in the difference between the price of the stock and price of the one-year call option, and this difference equals

$$S - C = PV(X) - P = 25e^{-0.10} - 5 \approx 17.62.$$

Answer A.
May 2006 Society of Actuaries Course 6 Examination, Problem No. 3b
You are given the following information for a bond portfolio:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Term (Years)</th>
<th>Annual Coupon Rate (Paid Semi-Annually)</th>
<th>Price</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>0.0%</td>
<td>122</td>
<td>200</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>4.8%</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>Floating equal to one-year Treasury Bill rate plus 50 bps reset annually</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

The floating rate has just been reset based on the one-year Treasury Bill rate of 4% (bond equivalent yield). Calculate the effective duration of the portfolio.

A. 4.74  B. 3.50  C. 2.90  D. 2.24  E. 1.92

Solution.
Bond A is a zero-coupon bond maturing in 10 years. Its Macaulay duration is 10 years. Its effective duration is the Macaulay duration divided by $1 + i$, where $i$ is an appropriate effective annual interest rate. Because this bond matures at 200, and its current price is 122, the effective annual interest rate for it is:

$$i_A = \left(\frac{200}{122}\right)^{\frac{1}{10}} - 1 \approx 5.067166\%.$$  

Therefore, its effective duration is approximately

$$\frac{10}{1 + i_A} \approx \frac{10}{1.05067166} = 9.5177213.$$  

Bond B is a coupon bond trading at par, and therefore, its yield rate is equal to the coupon rate of 2.4% semi-annually. This is equivalent to the annual effective interest rate of

$$i_B = 1.024^2 - 1 \approx 4.8576\%.$$  

Its Macaulay duration is

$$\frac{0.5 \cdot \frac{2.4}{1.024} + 1 \cdot \frac{2.4}{1.024^2} + 1.5 \cdot \frac{2.4}{1.024^3} + 2 \cdot \frac{2.4}{1.024^4} + 2.5 \cdot \frac{2.4}{1.024^5} + 3 \cdot \frac{102.4}{1.024^6}}{100} \approx 2.82961626.$$  

Its duration is

$$\frac{2.82961626}{1.024^2} \approx 2.69853235.$$  

Bond C is based on the Treasury Bill bond equivalent yield of 4%. For Treasury Bills, bond equivalent yield is defined as

$$\text{Bond Equivalent Yield} = \frac{\text{Face Value} - \text{Price}}{\text{Price} \cdot \frac{365}{n}},$$  

where $n$ is the number of days remaining till Treasury Bill's maturity. The price of a Treasury Bill is the present value of its maturity value based on its effective yield. Since this is an annual Bill, $n = 365$. Therefore, assuming 10000 face value,
\[
4\% = \frac{10000 - \text{Price}}{\text{Price}},
\]
so that
\[
\text{Price} \cdot 1.04 = 10000,
\]
and this tells us that the effective annual interest rate on a Treasury Bill is 4\%, and on Bond C, it is 4.50\%, because this bond’s rate is T-Bill plus 50 basis points. It produces a cash flow of 4.5 plus 100 (reset value) at time 1, so its Macaulay duration is 1, and its effective duration is
\[
\frac{1}{1.045} \approx 0.9569378.
\]
Note that the total price, i.e., market value, of the entire portfolio is 322. Effective duration of the entire portfolio is a market-value weighted average of the effective durations of the pieces of the portfolio:
\[
\frac{122}{322} \cdot 9.5177213 + \frac{100}{322} \cdot 2.69853235 + \frac{100}{322} \cdot 0.9569378 = 4.74133234.
\]
Answer A.
You are given the following:
• Strip bond at 1000 par value.
• Cash flows are payable at the end of the year.
• Annual rates of return.

<table>
<thead>
<tr>
<th>Year</th>
<th>Current price of strip bond</th>
<th>Spot rate</th>
<th>Forward rate</th>
<th>Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>900</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>9.0%</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>6.5%</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>720</td>
<td>5.0%</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Calculate the present value of the cash flows.

A. 148  B. 153  C. 155  D. 158  E. 163

Solution.
Let \( s_i \) stand for the spot rate for the period of time ending at the end of year \( i \), for \( i = 1, 2, 3, 4, 5 \), and let \( f_i \) be the forward rate for the period of time \([i−1, i] \), for \( i = 1, 2, 3, 4, 5 \). The present value sought equals

\[
\frac{100}{(1 + s_3)^3} + \frac{100}{(1 + s_4)^4}.
\]

But we do not know the two spot rates used. However, we do know the forward rate \( f_3 = 6.5\% \), and the spot rate \( s_2 = 9.0\% \). Thus

\[(1 + s_3)^3 = (1 + s_2)^3 \cdot (1 + f_3) = 1.09^3 \cdot 1.065 = 1.2653265.
\]

We also know that

\[720 \cdot (1 + s_5)^5 = 1000,
\]

and \( f_5 = 5\% \), and combining these two pieces of information gives us

\[
(1 + s_4)^4 = \frac{(1 + s_2)^5}{1 + f_5} = \frac{1000}{720 \cdot 1.05} = \frac{1000}{720 \cdot 1.05} \approx 1.32275132.
\]

Therefore

\[
\frac{100}{(1 + s_3)^3} + \frac{100}{(1 + s_4)^4} = \frac{100}{1.09^2 \cdot 1.065} + \frac{100}{720 \cdot 1.05} = 154.63.
\]

Answer C.
May 2006 Society of Actuaries Course 6 Examination, Problem No. 7
You are given the following with respect to a security:
• Market value: 1000.
• Cash flow at end of year 1: 500.
• Cash flow at end of year 2: 700.
Calculate the modified duration.

Solution.
Let us write \( i \) for the yield to maturity of this security. Then
\[
1000 = \frac{500}{1 + i} + \frac{700}{(1 + i)^2},
\]
and therefore
\[
1000(1 + i)^2 - 500(1 + i) - 700 = 0.
\]
This gives
\[
1 + i = \frac{500 \pm \sqrt{500^2 - 4 \cdot 1000 \cdot (-700)}}{2 \cdot 1000} = \begin{cases} 1.12321246, \\ -0.62321246. \end{cases}
\]
Only the positive answer is feasible, and we conclude that \( i \approx 12.321246\% \).

The Macaulay duration of the first cash flow is 1, and the Macaulay duration of the second cash flow is 2. Therefore, the modified duration of the entire security is
\[
\frac{1}{1000} \begin{pmatrix} \frac{500}{1.12321246} & 1 & \frac{700}{1.12321246^2} \\ \frac{1.12321246}{1.12321246} & \frac{1.12321246}{1.12321246} & \frac{2}{1.12321246} \end{pmatrix} \approx 1.384286855.
\]
You are given the following with respect to a security:

<table>
<thead>
<tr>
<th>Shift in Yield Curve (basis points)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50</td>
<td>4,000</td>
</tr>
<tr>
<td>-25</td>
<td>3,600</td>
</tr>
<tr>
<td>0</td>
<td>3,400</td>
</tr>
<tr>
<td>+25</td>
<td>3,250</td>
</tr>
<tr>
<td>+50</td>
<td>3,100</td>
</tr>
</tbody>
</table>

• Accrued interest: 900.
• Maturity value: 10,000.

Calculate the effective duration using an option-adjusted spread model for a total yield curve shift of 50 basis points.

A. 5.5  B. 16.3  C. 20.6  D. 41.9  E. 52.9

Solution.
Recall that the quoted price does not include accrued interest. Total value of a bond, i.e., the flat price, equals the sum of the quoted price and the accrued interest. Therefore, the total value under current interest rate is $3400 + 900 = 4300$, while with rates increasing by 50 bps the total value will be $3100 + 900 = 4000$, and with rates decreasing by 50 bps the total value will be $4000 + 900 = 4900$. This gives the effective duration as approximately equal to

\[
D \approx \frac{P(i - \Delta i) - P(i + \Delta i)}{2P(i)(\Delta i)} = \frac{4900 - 4000}{2 \cdot 4300 \cdot 0.0050} \approx 20.93023256.
\]

Answer C.
Spring 2006 Society of Actuaries Course 6 Examination, Problem No. 28

You are given the following with respect to a zero-coupon bond portfolio:

<table>
<thead>
<tr>
<th>Term (Years)</th>
<th>Spot Rate</th>
<th>Maturity Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.20%</td>
<td>2,500,000</td>
</tr>
<tr>
<td>10</td>
<td>4.80%</td>
<td>1,000,000</td>
</tr>
<tr>
<td>15</td>
<td>5.50%</td>
<td>5,000,000</td>
</tr>
<tr>
<td>20</td>
<td>6.00%</td>
<td>3,000,000</td>
</tr>
</tbody>
</table>

Calculate the yield to maturity of the portfolio.

A. 5.1%  B. 5.2%  C. 5.3%  D. 5.4%  E. 5.5%

Solution.
The price of this portfolio is
\[
\frac{2,500,000}{1.042^5} + \frac{1,000,000}{1.048^{10}} + \frac{5,000,000}{1.055^{15}} + \frac{3,000,000}{1.06^{20}} \approx 5,835,983.09.
\]
The yield to maturity is the solution of the equation
\[
\frac{2,500,000}{(1 + i)^5} + \frac{1,000,000}{(1 + i)^{10}} + \frac{5,000,000}{(1 + i)^{15}} + \frac{3,000,000}{(1 + i)^{20}} = 5,835,983.09.
\]
This can be solved using a financial calculator that has cash flow worksheet, e.g., BA II Plus), or by trying all five answers. It is simplest to calculate first the effective yield over 5 years, let us call it \( j \), so that
\[
\frac{2.5}{1 + j} + \frac{1}{(1 + j)^2} + \frac{5}{(1 + j)^3} + \frac{3}{(1 + j)^4} = 5.83598309.
\]
We enter cash flows of 2.5, 1, 5 and 3, and the price of 5.83598309 to get the internal rate of return of \( j = 30.0462\% \). Then
\[
i = \left(1 + j\right)^\frac{1}{5} - 1 \approx 5.394892\%.
\]
Answer D.
Spring 2006 Casualty Actuarial Society Course 8 Examination, Problem No. 21

You are given the following information:
• The 6-month (annual) interest rates in Japan and the United States are 3% and 6%, respectively, with continuous compounding.
• The spot price of the yen is $0.00856.
• The futures price for a contract deliverable in 6 months is $0.00912.

Calculation the present value of riskless arbitrage profit available, per 1 yen.

A. $0.00034  B. $0.00042  C. $0.00049  D. $0.00056  E. No arbitrage is possible

Solution.

The arbitrage-free futures price is $0.00856 \cdot e^{(0.06 - 0.03) \cdot \frac{1}{2}} \approx $0.00869. This means that a six-month futures price in the market of $0.00912 requires a payment of

$0.00912 - 0.00856 \cdot e^{\frac{1}{2} (0.06 - 0.03)} \approx $0.00912 - $0.00869 = $0.00043

in excess of the arbitrage-free price upon contract maturity. By buying the underlying with borrowed funds and shorting the futures, one can collect this difference as a riskless profit upon contract maturity. The present value of that profit is

$e^{-\frac{1}{2} \cdot 0.06} \left(0.00912 - 0.00856 \cdot e^{\frac{1}{2} (0.06 - 0.03)}\right) \approx $0.000418.

Answer B.
Spring 2006 Casualty Actuarial Society Course 8 Examination, Problem No. 19
(multiple choice answers added)

A one-year long forward contract is entered into when the stock price is $100 and the annual risk-free rate of interest is 6%, with continuous compounding for all maturities. The stock is expected to pay a dividend in three months and again in nine months. Six months from now, the price of the stock is $105 and the risk-free rate of interest is still 6% per annum. The value of the long position at this time is $3.99. Calculate the dividend per share, assuming that the dividend is the same at three and nine months.

A. 1.8750  B. 1.9205  C. 2.005  D. 2.1256  E. 2.2500

Solution.
Let us write $D$ for the dividend sought. Let $F_0$ for the arbitrage-free forward price at the time when the forward contract is entered into. Then, using the forward-spot parity, we obtain

$$F_0 = \left(100 - De^{-0.06 \frac{3}{12}} - De^{-0.06 \frac{9}{12}}\right)e^{0.06 - 1} = 100e^{0.06} - D\left(e^{0.045} + e^{0.015}\right).$$

The value of the position in half a year from the time the position is entered into is the excess of the value of the forward over the value of an arbitrage-free price forward at the same time. The arbitrage-free forward price at time 0.5 years is

$$F_{0.5} = \left(105 - De^{-0.06 \frac{3}{12}}\right)e^{0.06 - \frac{1}{2}} = 105e^{0.03} - De^{0.015}.$$

By holding the original forward, the investor buys at $F_0$, while the forward issued at time 0.5 years lets him/her purchase at $F_{0.5}$. The value of the original forward position is the difference of these purchase prices in favor of the investor, and that’s positive if $F_0 < F_{0.5}$, because this means he/she pays less than what the market demands, and negative if $F_0 > F_{0.5}$, but since the actual purchase happens half a year hence, the difference should be discounted, and the value at time 0.5 is

$$(F_{0.5} - F_0) \cdot e^{-0.06 \frac{1}{2}} = (F_{0.5} - F_0) \cdot e^{-0.03} = 3.99.$$

Therefore,

$$3.99e^{0.03} = (105e^{0.03} - De^{0.015}) - \left(100e^{0.06} - D\left(e^{0.045} + e^{0.015}\right)\right) =$$

$$= 105e^{0.03} - De^{0.015} - 100e^{0.06} + De^{0.045} + De^{0.015} = 105e^{0.03} - 100e^{0.06} + De^{0.045}.$$

This results in

$$D = e^{-0.045} \cdot (3.99e^{0.03} - 105e^{0.03} + 100e^{0.06}) =$$

$$= 100e^{0.015} - 101.01e^{-0.015} \approx 2.005149442.$$

Answer C.
A currency swap has a remaining life of 15 months. The swap involves exchanging annual euro interest for dollar interest. The principal amounts are also exchanged at the end of the life of the swap. You are given the following additional information:

- The swap involves exchanging interest at 11% on 25 million euro for interest at 8% on $30 million once a year.
- The term structure of interest rates in both Europe and the United States is currently flat.
- If the swap were negotiated today, the interest rates exchanged would be 8% in euros and 6% in dollars.
- All interest rates are quoted with annual compounding.
- The current exchange rate (dollars per euro) is 1.25.

Calculate the value of the swap (in dollars) to the party paying dollars.

Solution.

The value of the remaining payments in euros is (valued in euros)

\[
0.11 \cdot 25000000 \cdot \frac{1}{1.08^{12}} + 1.11 \cdot 25000000 \cdot \frac{1}{1.08^{15}} \approx 27902397.20.
\]

The value of the remaining payments in dollars is (valued in dollars)

\[
0.08 \cdot 30000000 \cdot \frac{1}{1.06^{12}} + 1.08 \cdot 30000000 \cdot \frac{1}{1.06^{15}} \approx 32489294.80.
\]

The value of the swap to the party paying dollars is

\[
1.25 \left( 0.11 \cdot 25000000 \cdot \frac{1}{1.08^{12}} + 1.11 \cdot 25000000 \cdot \frac{1}{1.08^{15}} \right) - \left( 0.08 \cdot 30000000 \cdot \frac{1}{1.06^{12}} + 1.08 \cdot 30000000 \cdot \frac{1}{1.06^{15}} \right) \approx 1.25 \cdot 27902397.20 - 32489294.80 \approx 2388701.70.
\]

Answer B.
Spring 2006 Casualty Actuarial Society Course 8 Examination, Problem No. 30
(multiple choice answers added)

Consider the following information from one year ago:
- Futures exchange rate was 0.80 pounds per dollar.
- Initial exchange rate was 0.75 pounds per dollar.
- British risk-free rate was 9.0% per annum, continuously compounded.

One year ago, a U.S. investor invested in a one-year risk-free British government bill. Also at that time, the investor hedged the exchange rate risk in the investment by using a futures contract. Calculate the dollar-denominated risk-free continuously compounded interest rate that the investor locked into one year ago.

A. 2.55%  
B. 2.75%  
C. 2.90%  
D. 3.25%  
E. 3.50%

Solution.
Consider the entire transaction as performed in U.S. dollars, and view the British pound as an interest-bearing instrument on which a futures contract is purchased. The spot price of a British pound is \( \frac{1}{0.75} = \frac{4}{3} \) dollars. The futures price of a British pound is \( \frac{1}{0.80} = \frac{5}{4} \) dollars. Let \( r \) be the unknown risk-free rate on the U.S. dollar. Using the futures-spot parity we obtain
\[
\frac{5}{4} = \frac{4}{3} \cdot e^{(r - 0.09)}.
\]
This gives
\[
r = \ln \left( \frac{15}{16} \cdot e^{0.09} \right) \approx 2.546148\%.
\]
Answer A.
Consider the following information about European options on stock ABC:

- Strike price is $95.
- Current stock price is $100.
- Time to expiration is 2 years.
- The stock pays no dividends.
- Price of a put is $0.75.

Calculate the price of the call on stock ABC with strike price $95 if the continuously compounded risk-free rate is 5%.


Solution.

Let $C$ denote the price of the call option sought, $X = 95$ be the strike price, $S = 100$ be the current price of the underlying, $t = 2$ be the time to expiration of options considered, $P = 0.75$ be the price of the put option, and $\delta = 5\%$ be the risk-free force of interest. The put-call parity formula is

$$C - P = S - PV(X).$$

Substituting known values, we obtain

$$C - 0.75 = 100 - 95e^{-0.05 \cdot 2}.$$

This gives

$$C = 100 - 95e^{-0.10} + 0.75 = 14.79.$$

Answer A.
Society of Actuaries May 2006 Course 6 Examination, Problem No. 28

You are given the following with respect to a zero-coupon bond portfolio:

<table>
<thead>
<tr>
<th>Term (Years)</th>
<th>Spot Rate</th>
<th>Maturity Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.20%</td>
<td>2,500,000</td>
</tr>
<tr>
<td>10</td>
<td>4.80%</td>
<td>1,000,000</td>
</tr>
<tr>
<td>15</td>
<td>5.50%</td>
<td>5,000,000</td>
</tr>
<tr>
<td>20</td>
<td>6.00%</td>
<td>3,000,000</td>
</tr>
</tbody>
</table>

Calculate the yield to maturity of the portfolio.

A. 5.1%  B. 5.2%  C. 5.3%  D. 5.4%  E. 5.5%

Solution.

The price of this portfolio is

\[
\frac{2500000}{1.042^5} + \frac{1000000}{1.048^{10}} + \frac{5000000}{1.055^{15}} + \frac{3000000}{1.06^{20}} \approx 5835983.09.
\]

The yield to maturity is the solution of the equation

\[
\frac{2500000}{(1+i)^5} + \frac{1000000}{(1+i)^{10}} + \frac{5000000}{(1+i)^{15}} + \frac{3000000}{(1+i)^{20}} = 5835983.09.
\]

This can be solved using a financial calculator that has cash flow worksheet, e.g., BA II Plus), or by trying all five answers. It is simplest to calculate first the effective yield over 5 years, let us call it \( j \), so that

\[
\frac{2.5}{1+j} + \frac{1}{(1+j)^5} + \frac{5}{(1+j)^{10}} + \frac{3}{(1+j)^{15}} = 5.83598309.
\]

We enter cash flows of 2.5, 1, 5 and 3, and the price of 5.83598309 to get the internal rate of return of \( j \approx 30.0462\% \). Then

\[
i = \left(1 + j\right)^{\frac{1}{5}} - 1 \approx 5.394892\%.
\]

Answer D.
A financial institution pays 6-month LIBOR and receives 7% per year with semiannual compounding on a swap with notional principal of $50 million. The remaining payment dates are in 4, 10, and 16 months. The LIBOR rates with continuous compounding for 4, 10, and 16 month maturities are 8%, 8.5%, and 9%, respectively. The 6-month LIBOR rate at the last payment date was 8.8% with semiannual compounding. Calculate the value of the swap to the financial institution.

A. –$1.6 million  B. –$0.6 million  C. 0  D. $0.5 million  E. $1.8 million

Solution.
The floating payment of the swap is based on the 6-month LIBOR, so to establish the value of the swap now we use current value of that 6-month LIBOR. That value is now 4.4% per half year. The fixed side payment is 3.5% per half year. Both are based on $50 million principal. But the floating side will reset in the next payment, so its market value at the reset will be exactly the principal, while the fixed side will pay the principal at maturity. So the floating side amounts to 4.4% of $50 million, or $2.2 million, plus $50 million principal, paid on 4 months, while the fixed side amounts to payments of 3.5% of $50 million, or $1.75 million, paid in 4 months, 10 months, and 16 months, together with principal of $50 million paid in 16 months. Using LIBOR rates with continuous compounding we calculate the value of the swap, in $million, as:

$$-52.2 \cdot e^{-0.08 \cdot \frac{4}{12}} + \left( 1.75e^{-0.08 \cdot \frac{4}{12}} + 1.75e^{-0.085 \cdot \frac{10}{12}} + 51.75e^{-0.09 \cdot \frac{16}{12}} \right) \approx -1.5940.$$ 

Answer A.
You are given the following information:
• Strike price is $30.
• Current stock price is $29.
• Dividends of $1.50 are expected to be paid after 3, 5, and 7 months.
• Continuously compounded risk-free interest rate is 10% per annum.
• Price of 6-month European call with a strike price of $30 is $2.

Calculate the price of a European put option that expires in 6 months.

A. 1.54  B. 2.90  C. 3.57  D. 4.03  E. 4.44

Solution.
Put-call parity says

\[
\text{Put} = \text{Call} + \text{PV(Exercise Price)} - \text{Ex-dividend Stock Price} = 2 + 30e^{-0.10 \cdot \frac{1}{2}} - \left(29 - 1.5e^{-0.10 \cdot \frac{3}{12}} - 1.5e^{-0.10 \cdot \frac{5}{12}}\right) \approx 4.4386.
\]

Answer E.
You have created a bear spread from call options with strike prices of $20 and $25 that cost $4 and $1, respectively. The call options expire on the same day. Draw the profit diagram for the bear spread.

A.

B.

C.
Solution.
Bear spread created with calls is defined as a short position in a lower exercise price call and a long position in a higher exercise price call. In this case, short 20 call, bringing in premium income of 4, and long 25 call, at a cost of 1. The payoff, as a function of the price of the underlying $S$, is

$$4 - 1 = 3 \text{ for } S < 20,$$
$$3 + 20 - S = 23 - S \text{ for } 20 \leq S < 25,$$
$$23 - S + S - 25 = -2 \text{ for } S \geq 25.$$

This formula is represented in the graph in answer E.
Answer E.
You are given the following information:

• An American investor purchases 1000 shares of a Canadian company on January 1, 2006 for $27.50 Canadian (CAD) per share.
• The investor sells all shares for $30 CAD on December 31, 2006.
• The exchange rate was $1.10 CAD per U.S. dollar (USD) in January 1, 2006.
• The exchange rate was $1.20 CAD per USD on December 31, 2006.

Calculate the investor’s rate of return in U.S. funds on December 31, 2006.

A. 0%    B. 0.50%    C. 1.00%    D. 1.50%    E. 2.00%

Solution.
The investor spent $27500 Canadian on the purchase, and at that time, that amounted to $25000. At the time of the sale, the shares were worth $30000 Canadian, and that was worth $25000 U.S. Therefore, the rate of return was 0%.
Answer A.
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 9
You are given the following information about a bond:
• The par value is $1,000.
• The current price is $950.
• The number of years to maturity is 3 years.
• The annual coupon is 5%.
• Coupons are paid semiannually.
Assume all interest is compounded semiannually. At the end of Year 2, the bond will have a yield-to-maturity of 5%. Calculate the realized annual effective yield for an investor with a 2-year holding period and a reinvestment rate of 4% over the period.

A. 7.61%   B. 7.75%   C. 7.89%   D. 7.98%   E. 8.10%

Solution.
At the end of Year 2, the bond will have two semi-annual coupons of $25 remaining, and a principal repayment of $1000 remaining, and its market value at then yield to maturity of 5% (actually, 2.5% per half year) will be $1000, as the bond will be trading at par. Over those two years, the bond will pay four semi-annual coupons of $25 that will accumulate at 2% semiannual reinvestment rate to $25 \cdot s_{\frac{2}{4}^{\%}} = $103.04. Thus the investor paid $950 at time 0 and received $103.04 + $1000 = $1103.04 at time 2. We need to find the annual effective rate earned by the investor. Let us denote it by \( j \). We have:

\[
950 \cdot (1 + j)^2 = 1103.04,
\]
so that

\[
j = \left( \frac{1103.04}{950} \right)^{\frac{1}{2}} - 1 \approx 7.7541\%.
\]
Answer B.
You are given the following information about a May futures contract:

<table>
<thead>
<tr>
<th>Date</th>
<th>Futures settlement price</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 7</td>
<td>$483</td>
</tr>
<tr>
<td>April 8</td>
<td>$475</td>
</tr>
<tr>
<td>April 9</td>
<td>$475</td>
</tr>
<tr>
<td>April 10</td>
<td>$477</td>
</tr>
</tbody>
</table>

On April 7, an investor contracted to buy three futures contracts at $485 per ounce, with a contract size of 100 ounces. The initial margin is $2500 per contract. Calculate the value of the margin account at the end of trading on April 7. Assume no interest is earned on the margin account balance.

A. $8100       B. $7800   C. $7500    D. $7200    E. $6900

Solution.

At $485 per ounce, with a contract size of 100 ounces, and three contracts, the initial value is $145,500. Initial margin is three times $2500, i.e., $7500. Contract declines in value by $2, and the total change is $2 times 100 ounces, times 3 contracts, for a decline of $600, so new margin account balance is $6900.

Answer E.
You are given the following information about a May futures contract:

<table>
<thead>
<tr>
<th>Date</th>
<th>Futures settlement price</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 7</td>
<td>$483</td>
</tr>
<tr>
<td>April 8</td>
<td>$480</td>
</tr>
<tr>
<td>April 9</td>
<td>$475</td>
</tr>
<tr>
<td>April 10</td>
<td>$477</td>
</tr>
</tbody>
</table>

On April 7, an investor contracted to buy three futures contracts at $485 per ounce, with a contract size of 100 ounces. The initial margin is $2500 per contract. Assume no interest is earned on the margin account balance. Assume also that the maintenance margin is 75% of the initial margin. Determine the date when a margin call will be made and the amount of the margin call. Margin call will demand restoring original initial margin.

A. April 8, for $3000
B. April 9, for $3000
C. April 8, for $1125
D. April 9, for $3125
E. No margin call will be made

Solution.
The maintenance margin is 75% of $7500, i.e., $5625. The investor has a loss of $2 times 100 ounces, times 3 contracts, i.e., $600, on April 7, and the margin balance is then $6900, sufficient. On April 8, there is a decline of $3 times 100 ounces, times 3 contracts, for a total of $900, and the margin account balance is $6000, still sufficient. On April 9, there is a $5 decline, and the margin account balance becomes $6000 – $1500 = $4500. To restore the original balance, a margin call of $3000 is needed.

Answer B.
You are given the following information about a jewelry manufacturer:
• On February 22, the manufacturer expects to acquire 20,000 troy ounces of silver in July.
• The spot price of silver on February 22 is 600 cents per troy ounce.
• The futures price for July delivery is 590 cents per troy ounce.
• There is no daily settlement.
• Each futures contract is for the delivery of 5,000 troy ounces of silver.

The jewelry manufacturer hedges the exposure by buying four contracts for July delivery of silver. At maturity, futures are settled financially, without actual delivery. Assuming no transaction costs, find the gain or loss on July 15 of this hedging strategy if the spot price of silver on July 15 is 585 cents per troy ounce.

A. Loss of $100,000  B. Loss of $1,000  C. No gain or loss  D. Gain of $1,000  E. Gain of $1,000,000

Solution.
Because there is no daily settlement, the futures contract works just like a forward. Because of no transaction costs, the only cash flows happen at contract maturity, when the spot price is 585 cents per ounce, and the contract price is 590 cents per ounce. So the manufacturer effectively pays 590 cents per ounce for something that costs 585 cents per ounce, a loss of 5 cents per ounce, or $250 per contract, or $1,000 loss for 4 contracts. Answer B.
You are given the following information:

- The current price of assets underlying a futures contract is $50.
- The time to delivery date is 2 years.
- The risk-free interest rate is 5% annual effective.
- The futures contract delivery price is $60.

Assume that the asset provides income with present value of $10. Calculate the value of a long forward contract.


Solution.
The prepaid forward price of the underlying is $50 – $10 = $40. If the forward contract were to be entered into at no cost, the contract price would be $40 \cdot 1.05^2 \approx $44.10. Instead, the price is $60, which means that the party long the contract purchases at $60 instead of $44.10, overpaying $15.90 at contract expiration. This is equivalent to overpayment now of $15.90 \cdot 1.05^{-2} \approx $14.42. But that value is, of course, negative to the long party.

Answer B.
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 21

You are given the following information about a swap between a financial institution and Company A:

• The notional principal is $200,000,000.
• The financial institution pays 6-month LIBOR.
• Company A pays 7% per annum (with semiannual compounding).
• The swap’s remaining life is 9 months.
• The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 9.0%, 9.25%, and 9.5%, respectively.
• The 6-month LIBOR rate at the last payment date was 9.1% (with semiannual compounding).

Calculate the value of the swap from the perspective of the financial institution, in millions of dollars. *Explanatory note:* Assume the floating side payments are based on the floating rate in arrears (i.e., 6 months before the payment) and for any future floating side payment, use the appropriate forward rate.

A. 4.5  B. 3.25  C. 1.5  D. –3.25  E. –4.5

Solution.

Remaining swap payments will occur in 3 months and 9 months. In order to establish the value of the swap, we need to find the floating side rate used in the swap for payments happening in 3 months and 9 months. The 6-month LIBOR rate at the last payment date was 9.1%, with semiannual compounding, and that is the rate used for the payment happening in 3 months, i.e., half of 9.1% = 4.55% is the floating rate for payment in 3 months. For payment in 9 months, we need to find the forward rate from time 3 months to time 9 months based on the given LIBOR rates. Since the 3 month LIBOR is 9% continuously compounded and 9 month LIBOR is 9.25% continuously compounded, we have, if we write $f_{0.25,0.75}$ for the forward rate sought, expressed as an effective rate over 6 months, we have

$$e^{0.075 \cdot 0.0925} = e^{0.25 \cdot 0.09} \cdot (1 + f_{0.25,0.75}),$$

and therefore

$$f_{0.25,0.75} = \frac{e^{0.075 \cdot 0.0925}}{e^{0.25 \cdot 0.09}} - 1 \approx 4.7991\%.$$

The remaining payments (in millions of dollars) are:

<table>
<thead>
<tr>
<th>Time</th>
<th>Company A pays</th>
<th>Financial institution pays</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 months</td>
<td>3.5% of 200 = 7</td>
<td>4.55% of 200 = 9.1</td>
</tr>
<tr>
<td>9 months</td>
<td>3.5% of 200 = 7</td>
<td>4.7991% of 200 = 9.5982</td>
</tr>
</tbody>
</table>

Using the LIBOR rates given, we calculate the value of the swap to the financial institution as

$$(7 - 9.1) \cdot e^{-0.25 \cdot 0.09} + (7 - 9.5982) \cdot e^{-0.75 \cdot 0.0925} \approx -4.4773.$$

Answer E.
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 30c

A 3-year $100 bond with a yield of 12% continuously compounded pays a 6% coupon at the end of each year. Using this bond’s duration estimate the effect of 0.3% decrease on the bond’s yield on the bond price.

A. −0.63  B. 0.63  C. 0.71  D. 6.30  E. 7.10

Solution.
The bond’s price is
$$6e^{-0.12} + 6e^{-2 \cdot 0.12} + 106e^{-3 \cdot 0.12} \approx 83.99.$$  
The bond’s Macaulay duration is
$$\frac{6 \cdot 1 \cdot e^{-0.12} + 6 \cdot 2 \cdot e^{-2 \cdot 0.12} + 106 \cdot 3 \cdot e^{-3 \cdot 0.12}}{6e^{-0.12} + 6e^{-2 \cdot 0.12} + 106e^{-3 \cdot 0.12}} \approx 2.8171.$$  
The duration is
$$\frac{2.8171}{e^{0.12}} \approx 2.4985.$$  
The change in the price estimated with the use of duration is
$$\Delta P \approx -D \cdot P \cdot \Delta i \approx -2.4985 \cdot 83.99 \cdot (-0.003) = 0.63.$$  
Answer B.
Spring 2008 Casualty Actuarial Society Course 8 Examination, Problem No. 31

An insurance company has an obligation to make a payment of $15,485 eight years from today. The market interest rate is 11%. The insurer plans to fund the obligation using zero-coupon bonds that mature in four years and perpetuities (immediate) that pay annual coupons. Determine the amounts to be invested today by the insurer in perpetuities and zero-coupon bonds to immunize the obligation.

Solution.
The asset portfolio will need to have the same Macaulay duration as the liability, i.e., 8. Let $w$ be the fraction of the asset portfolio invested in the four-year zero coupon bonds. Recall that the Macaulay duration of a level perpetuity immediate is the price of a unit perpetuity due. The Macaulay duration of the asset portfolio is

$$w \cdot 4 + (1 - w) \cdot \frac{1}{d_{11\%}} = w \cdot 4 + (1 - w) \cdot \frac{1.11}{0.11} = 4w + \frac{111}{11} - \frac{111}{11}w = \frac{111}{11} - \frac{67}{11}w.$$ 

Therefore, 

$$8 = \frac{111}{11} - \frac{67}{11}w$$

resulting in $w = \frac{23}{67}$. The present value of the liability is

$$15,485 \cdot 1.11^8 \approx 6,719.35.$$ 

Of this, 

$$\frac{23}{67} \cdot 6,719.25 \approx 2,306.61$$ 

should be invested in four-year bonds and 

$$6,719.35 - 2,306.61 = 4,412.74$$ 

in perpetuities.
Given the following information about a 10-year corporate bond purchased on January 1, 2011:

- The par value is $1000.
- The coupon rate is 6%.
- Coupons are paid annually on December 31.
- The market interest rate on January 1, 2013 is 4%.

Calculate the current yield of the bond on January 1, 2013.

A. 4.00%   B. 4.77%   C. 5.29%   D. 5.87%   E. 6.00%

Solution.
The price of the bond on January 1, 2013 is

\[ 60a_{\frac{4}{10}} + 1000 \cdot 1.04^{-8} = 1134.65. \]

Using BA Plus Pro this is calculated by entering: 60, PMT, 1000, FV, 8, N, 4, I/Y, CPT, PV, with the result being negative because the calculator treats the process as paying off that negative balance. The current yield is defined as the ratio of the coupon to the price, i.e.,

\[ \frac{60}{1134.65} = 5.2880\%. \]

Answer C.
Given the following information about Treasury Bonds:

<table>
<thead>
<tr>
<th>Bond Price</th>
<th>Years to Maturity</th>
<th>Annual Coupon Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100.90</td>
<td>1.0</td>
<td>4%</td>
</tr>
<tr>
<td>$104.80</td>
<td>1.0</td>
<td>8%</td>
</tr>
<tr>
<td>$100.80</td>
<td>1.5</td>
<td>4%</td>
</tr>
<tr>
<td>$101.68</td>
<td>2.0</td>
<td>4%</td>
</tr>
</tbody>
</table>

Coupons are paid semi-annually. The par value is $100. Calculate the 1-year forward rate with annual compounding at the end of year 1.

A. 3.00%  B. 3.09%  C. 3.19%  D. 3.29%  E. 3.39%

Solution.

Let us write \( s_{0.5} \) for the spot rate from time 0 to time 0.5, expressed as annual effective rate. Similarly, let \( s_1 \) be the spot rate from time 0 to time 1, expressed as annual effective, and \( s_{1.5} \), \( s_2 \) be the annual effective spot rates from time 0 to time 1.5 and 2, respectively.

We have, based on the prices of bonds given

\[
100.90 = \frac{2}{(1 + s_{0.5})^{0.5}} + \frac{102}{(1 + s_1)^1},
\]

\[
104.80 = \frac{4}{(1 + s_{0.5})^{0.5}} + \frac{104}{(1 + s_1)^1},
\]

\[
100.80 = \frac{2}{(1 + s_{0.5})^{0.5}} + \frac{2}{(1 + s_1)^1} + \frac{102}{(1 + s_{1.5})^{1.5}},
\]

\[
101.68 = \frac{2}{(1 + s_{0.5})^{0.5}} + \frac{2}{(1 + s_1)^1} + \frac{2}{(1 + s_{1.5})^{1.5}} + \frac{102}{(1 + s_2)^2}.
\]

If we subtract half of the second equation from the first equation, we obtain

\[
48.5 = \frac{50}{1 + s_1},
\]

so that \( 1 + s_1 = \frac{50}{48.5} = \frac{100}{97}. \) From the first equation, we also get

\[
100.90 = \frac{2}{(1 + s_{0.5})^{0.5}} + 102 \cdot \frac{97}{100},
\]

or \( (1 + s_{0.5})^{0.5} = \frac{1}{0.98}. \) Note also that we have in the process established that

\[
\frac{2}{1 + s_1} = 2 \cdot 0.97 = 1.94 \quad \text{and} \quad \frac{2}{(1 + s_{0.5})^{0.5}} = 2 \cdot 0.98 = 1.96,
\]

so that

\[
\frac{2}{(1 + s_{0.5})^{0.5}} + \frac{2}{1 + s_1} = 1.96 + 1.94 = 3.9.
\]

The last two of the initial four equations become
\[ 100.80 = \frac{2}{(1 + s_{0.5})^{0.5}} + \frac{2}{(1 + s_1)^{1}} + \frac{102}{(1 + s_{1.5})^{1.5}} = 3.9 + \frac{102}{(1 + s_{1.5})^{1.5}}, \]

and
\[ 101.68 = 3.9 + \frac{2}{(1 + s_{1.5})^{1.5}} + \frac{102}{(1 + s_2)^{2}}. \]

The first of these two gives us
\[ (1 + s_{1.5})^{1.5} = \frac{102}{100.80 - 3.9} = \frac{102}{96.9} \approx 1.0526, \]

and the second one results in
\[ (1 + s_2)^2 = \frac{102}{101.68 - 3.9 - \frac{2 \cdot 96.9}{102}} = \frac{102}{95.88} \approx 1.0638. \]

The forward rate sought \( f_{1,2} \) is then calculated from the identity
\[ (1 + s_2)^2 = (1 + s_1)(1 + f_{1,2}) \]

so that
\[ f_{1,2} = \frac{(1 + s_2)^2}{1 + s_1} - 1 = \frac{102}{95.88} - 1 = 1.02 \cdot \frac{97}{95.88} - 1 \approx 3.1915\%. \]

Answer C.
Spring 2009 Casualty Actuarial Society Course 8 Examination, Problem No. 17

Given the following information about a 2-year zero-coupon bond:

- The current price is $1050.
- The time to maturity of the bond is 9 months.
- The current risk-free rate is 4.35%, annual, continuously compounded.

Company ABC plans to enter into a forward contract to sell the bond in 5 months for $1100. Describe a series of transactions that an arbitrageur could enter into and realize a profit and calculate the amount of profit. Note from K.O.: Assume no transaction costs, and in particular assume that the short forward position to sell the bond in 5 months for $1100 is costless to enter into, and an arbitrageur borrows and lends at the risk-free rate. The value of the profit should be calculated at the moment when the transaction is closed.

A. Short the bond, enter into a long forward position, earn profit of $20.30
B. Short the bond, enter into a long forward position, earn profit of $30.80
C. Short forward, short bond, buy a call on the bond with exercise price of $1100 and buy a put on the bond with exercise price of $1050, earn profit of 2.50
D. Short forward, buy the bond, earn profit of $30.80
E. Short forward, buy the bond, earn profit of $2.50

Solution.

The forward price for a contract maturing in 5 months is $1050 \cdot e^{0.0435 \frac{5}{12}} \approx $1069.20. But the price of the contract that ABC is entering into is $1100, so the price of the contract is in excess of an arbitrage-free price. The forward is too expensive in relation to the underlying, and the underlying is too cheap in relation to the forward. An arbitrageur should buy the cheaper and sell the more expensive of the two assets. The transactions should buy the bond and short the forward, and the transaction is:

- Short forward at $1100 price.
- Buy the bond for $1050 with borrowed funds and pay 4.35%, annual, continuously compounded, for the funds.
- In 5 months, deliver the bond to cover the commitment under the short forward, and be paid $1100 for the bond.
- Use the $1100 to pay off the loan with $1069.20 of it, and keep the rest, i.e., $30.80.

Answer D.
Spring 2010 Casualty Actuarial Society Course 8 Examination, Problem No. 6

Given the following information:

- A ten-year 5% annual coupon bond was purchased on January 2, 2010 with a 7% yield to maturity.
- The bond was later sold on January 2, 2011, immediately after coupon payment, at a yield to maturity of 6%.
- Tax rates on capital gains and interest income are 20% and 30%, respectively.
- Interest rates are compounded annually.

Calculate the after-tax holding period return.

A. 10.00%  
B. 10.33%  
C. 10.50%  
D. 10.70%  
E. 11.00%

Solution.

For simplicity, let us assume that the par value of the bond is $100. The purchase price of the bond was

$$\frac{5}{10}\% + 100 \cdot 1.07^{-10} \approx \$85.95.$$  

Using the BA II Plus calculator, we would calculate this by entering 

PMT 5, N 10, I/Y 7, FV 100, CPT PV 

and the answer would be given as negative, because the calculator treats this as a loan being paid off. The price at which the bond is sold is

$$\frac{9}{9}\% + 100 \cdot 1.06^{-9} \approx \$93.20.$$  

The investor receives a coupon payment of $5 and a capital gain of

$$93.20 - 85.95 = \$7.25,$$

for a total of $12.25. The tax rules assume that the interest rate earned is the yield as of the date of purchase, i.e., 7% on $85.95, i.e., $6.02, and that is taxed at 30%, leaving 70% to the investor, i.e., $4.21. The rest, $12.25 - $6.02 = $6.23, is taxed at 20%, leaving 80% to the investor, i.e., $4.98. The investor collects a total of $4.21 + $4.98 = $9.19 on an investment of $85.95, and the after-tax rate of return is

$$\frac{9.19}{85.95} \approx 10.6923\%.$$  

Answer D.
Spring 2010 Casualty Actuarial Society Course 8 Examination, Problem No. 11b

Given the following information:

- The table below describes the rates available for both Company A and Company B.

<table>
<thead>
<tr>
<th></th>
<th>Company A</th>
<th>Company B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can borrow at a fixed rate of:</td>
<td>6.875%</td>
<td>5.375%</td>
</tr>
<tr>
<td>Can borrow at a floating rate of:</td>
<td>LIBOR + 1.65%</td>
<td>LIBOR + 0.90%</td>
</tr>
</tbody>
</table>

- Company A prefers a fixed interest rate.
- Company B prefers a floating interest rate.
- Company A and Company B can collect the other directly and create a swap, which benefits both companies equally. Calculate the resultant rate that Company A pays on its loan after the effect of the swap.

A. 5.375%  B. 5.50%  C. 5.95%  D. 6.50%  E. 6.875%

Solution.

Company B can borrow at a lower rate than Company A, whether they borrow at a fixed or a floating rate. However, the difference is 6.875% – 5.375% = 1.50% for the fixed rate, and 1.65% – 0.90% = 0.75% for the floating rate. Thus, Company A has a comparative advantage in borrowing at a floating rate, while Company B has a comparative advantage in borrowing at a fixed rate. Yet their preferences are opposite. So if they borrow according to their comparative advantage, and then swap, they will benefit. Company A borrows at LIBOR + 1.65% and Company B borrows at 5.375%.

The net comparative advantage is: 1.50% – 0.75% = 0.75%, and the companies should split it equally, 0.375% each, in the swap to have equal benefit. Thus the result should be that after swap:

- Company A should pay fixed rate of 6.875% – 0.375% = 6.50%,
- Company B should pay floating rate of LIBOR + 0.90% – 0.375% = LIBOR + 0.525%.

This gives us the answer as 6.50%. Just to understand things better, we will describe the swap. If we assume that the floating side pays LIBOR, the swap can be arranged as:

- Company A pays 6.50% – 1.65% = 4.85%,
- Company B pays LIBOR.

This means that the effective interest rate on its loan for Company A is:

\[ \text{LIBOR} + 1.65\% + 4.85\% - \text{LIBOR} = 6.50\%, \] as expected,

And the effective interest rate on its loan for Company B is:

\[ 5.375\% - 4.85\% + \text{LIBOR} = \text{LIBOR} + 0.525\%, \] as expected.

Answer D.
An investor would like to enter into a forward contract whereby in two years the investor exchanges a fixed amount of U.S. dollars for one million euros. Assume the current exchange rate is $1.50 per euro and that the continuously compounded risk-free interest rates are 2% in Europe and 1% in the United States. The investor can borrow and invest at the risk-free rate. Assume the two-year forward price for one million euros is now $1,475,000 and that the investor can either take a long or short position on a forward contract at this price. Determine if an arbitrage opportunity exists and if one does, calculate the present value of a profit in such an arbitrage opportunity involving one unit of the forward contract identified above.

A. There is no arbitrage opportunity
B. Less than $1500
C. At least $1500 but less than $3000
D. At least $3000 but less than $4500
E. $4500 or more

Solution.
The market price for a two-year forward contract to buy one million euros is $1,475,000, while the no arbitrage price is

\[ \$1,500,000 \cdot e^{-0.02 \cdot 2} \cdot e^{0.01 \cdot 2} = \$1,500,000 \cdot e^{-0.02} \approx \$1,470,298.01. \]

Thus, the market price is too high, and to take advantage of the arbitrage, one must short forward on one million euros, and buy one million euros without interest due on them with borrowed money. The one million euros without interest amounts to

\[ \€1,000,000 \cdot e^{-0.02 \cdot 2} \approx \€960,789.44, \]

and in order to purchase that amount at today’s exchange rate of $1.50 per euro, one must borrow \€960,789.44 \cdot 1.5 \$/\€ = \$1,441,184.16. In one year, the euros purchased will accumulate to one million euros, which will be sold through the forward contract for \$1,475,000. The loan taken in U.S. dollars will accumulate to

\[ \$1,441,184.16 \cdot e^{0.01 \cdot 2} \approx 1,470,298.01, \]

the no arbitrage price, as expected. The profit realized at time 2 is

\[ \$1,475,000 – \$1,470,298.01 = \$4,701.99 \]

and its present value is

\[ \$4,701.99 \cdot e^{-0.01 \cdot 2} \approx \$4608.88. \]

Answer E.
Given the following stock information:
• The current stock price is $60.
• The strike price is $57.
• The time to expiration is six months.
• The risk-free rate is 3.0% compounded continuously.
• European call option price is $3.
• Dividend amount to be paid in three months is $2.
• European put option price is $2.
Assume there are no transaction costs, it is possible to borrow or lend at the risk-free rate, and there are no taxes to consider. Use put-call parity to determine if an arbitrage opportunity exists, and if it does exists, calculate its value at the time of expiration of both options, per one unit of each option.

A. Less than $0.50
B. At least $0.50 but less than $0.75
C. At least $0.75 but less than $1.00
D. At least $1.00 but less than $1.50
E. $1.50 or more

Solution.
Put-call parity says that stock (without dividends) plus put must have the same price as call plus present value of exercise price. The price of one share of stock (without dividends) and one put option is

\[
\left(60 - 2 \cdot e^{-0.03 \cdot 0.25}\right) + 2 \approx 60.01.
\]

The price of one call option plus present value of exercise price is

\[
3 + 57 \cdot e^{-0.03 \cdot 0.5} \approx 59.15.
\]

As these are not equal, an arbitrage opportunity exists. To take advantage of it, one must buy the cheap portfolio (call plus present value of exercise price) and short the expensive one (stock plus put). The investor should write a put, and short the stock, receiving $62 up front. Buy a call with proceeds and invest the remaining $59 in a risk-free bond. In 3 months, the risk-free investment will have grown to $59 \cdot e^{0.03 \cdot 0.25} \approx 59.44$. At that time, a dividend of $2 must be paid on the short position in the stock, leaving $57.44 still invested in a risk-free bond. That amount accumulates to $57.88 at options’ expiration date. If the price of the stock at that time is less than $57, the put is exercised and the investor must purchase one share of stock for $57, covering the short position in stock with it. In that situation, the call is worthless, and the investor is left with $0.88. On the other hand, if the price of the stock is greater than $57, the put is worthless and the investor exercises the call, purchasing one share for $57. This share is used to cover the short position in stock, and the investor is left with $0.88 profit. Finally, if the stock price is exactly $57, both options are worthless, and the investor buys the stock in the market at $57 to cover the short, again left with profit of $0.88.

Answer C.
Spring 2010 Casualty Actuarial Society Course 8 Examination, Problem No. 21

Given the following information for an insurer’s property-casualty line of business for accident year 2009:

• The risk-free interest rate is 5%.
• The current yield on the insurer’s asset portfolio is 10%.
• The inflation rate is 0%.
• The loss payout pattern by development year is: Year 1 = 30%, Year 2 = 55%, Year 3 = 15%.
• Losses are paid in the middle of each year.
• Interest rates are compounded annually.

Calculate the Macaulay duration for the accident year 2009 loss reserves as of December 31, 2009. Note: The calculation should be done using the insurer’s rate of return as the interest rate.

A. 1.1 years  B. 1.3 years  C. 1.5 years  D. 1.7 years  E. 1.9 years

Solution.

If we assume $100 of losses are to be paid, $30 of that will be paid in 0.5 years, $55 of that will be paid in 1.5 years, and $15 of that will be paid in 2.5 years. Note that the amounts are not increased because inflation rate is 0%. The Macaulay duration is

\[
\frac{30 \cdot 0.5 \cdot 1.1^{-0.5} + 55 \cdot 1.5 \cdot 1.1^{-1.5} + 15 \cdot 2.5 \cdot 1.1^{-2.5}}{30 \cdot 1.1^{-0.5} + 55 \cdot 1.1^{-1.5} + 15 \cdot 1.1^{-2.5}} = 1.3094.
\]

Answer B.
Spring 2010 Casualty Actuarial Society Course 8 Examination, Problem No. 22a

An insurance company needs to make a payment of $1,967.15 in ten years. The insurance company would like to fund the obligation using four-year zero-coupon bonds and perpetuities paying annual coupons. The interest rate is 7% compounded annually. Calculate the amount the insurance company should invest in four-year zero-coupon bonds and perpetuities paying annual coupons in order to immunize the obligation from interest rate fluctuations.

A. Invest $468 in four-year zero-coupon bonds and $532 in perpetuities
B. Invest $485 in four-year zero-coupon bonds and $515 in perpetuities
C. Invest $500 in four-year zero-coupon bonds and $500 in perpetuities
D. Invest $515 in four-year zero-coupon bonds and $485 in perpetuities
E. Invest $532 in four-year zero-coupon bonds and $468 in perpetuities

Solution.
The present value of the liability is $1,967.15 \cdot 1.07^{-10} \approx $1,000.00. This means that the total value of the portfolio of assets backing this liability should also be set at $1,000. Additionally, we will match the Macaulay duration of assets and liability (that is, of course, equivalent to matching their durations). The Macaulay duration of a four-year zero-coupon bond is 4. The Macaulay duration of a level perpetuity immediate is
\[ \frac{1}{d} = \frac{1 + i}{i} = \frac{1.07}{0.07} = \frac{107}{7}. \]
If a fraction \( w \) of the asset portfolio is invested in the four-year zero-coupon bonds, then the Macaulay duration of the asset portfolio is
\[ w \cdot 4 + (1 - w) \cdot \frac{107}{7}, \]
and this quantity must equal 10. We solve the resulting equation
\[ 4w + \frac{107}{7} - \frac{107}{7}w = 10, \]
or
\[ \frac{107 - 28}{7}w = \frac{107 - 70}{7}, \]
resulting in \( w = \frac{37}{79} \) and \( 1 - w = \frac{42}{79} \). Thus, \( \frac{37}{79} \cdot $1,000 = $468.35 \) should be invested in four-year zero-coupon bonds, and \( \frac{42}{79} \cdot $1,000 = $531.65 \) should be invested in perpetuities. Answer A.