APPLICATION OF MEASURES OF FUZZINESS TO RISK CLASSIFICATION IN INSURANCE

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Abstract

We show how measures of fuzziness can be used to classify risks considered for insurance purposes, using the example of life insurance. A risk given is described as a fuzzy preferred risk, and then its fuzziness is measured to indicate its classification. We compare various measures of fuzziness, including the classical entropy measure of De Luca and Termini [2], the distance measure of Yager [9], and the axiomatic product measure (see e.g., Ebanks [3]), and we discuss their applicability to risk classification.

The problem

The theory of fuzzy sets developed by Zadeh [10, 11] has been successfully applied in modeling all forms of approximate reasoning, in particular in construction of fuzzy controllers designed to replace human experts. Until now, the most successful application of that theory came about in engineering, social sciences, medicine, computer science, and systems science (Klir and Folger [4]). Recently, there has been a new activity in the applications of fuzzy-set-theoretic methods - in the area of actuarial science. Buckley [1] gave a pioneering framework for the fuzzy mathematics of finance. Lemaire [5] calculated fuzzy set single premium for a pure endowment insurance, discussed a reinsurance problem characterized by its XL-deductible solved by applying decision-making techniques with fuzzy goals and constraints, and used fuzzy intersection to define a preferred risk.

Ostaszewski [6] provided a complete account of various methods of fuzzy set theory which are applicable in actuarial science. In particular, he showed how human experts making underwriting decisions could be replaced by fuzzy controllers constructed by modeling those experts' decision making process. He also showed, in the context of life insurance, that fuzzy clustering algorithms can be used to group risks considered for insurance.

Risk *classification*, which refers to categorizing risks for which the insurance is written, according to their probability of generating claims, and according to the sizes of those claims, is one of the fundamental concepts of actuarial science (Trowbridge [8]). Stiglitz [7] considers a basic economic model of insurance and shows that under equilibrium pricing, high risk group always purchases a different coverage than the low risk group. In fact, bundling of risks, or imperfections of market or information, leads to *antiselection* (Trowbridge [8]), i.e., the situation when low risk group opts out of the insurance contract in order to avoid paying the same premium as the high risk group.

This work is devoted to investigating an alternative method of classifying risks, based on the concept of *measure of fuzziness*.

The model

Lemaire [5] explains how the current underwriting practice in insurance is to use strict criteria for selection of preferred risk. For example, he points out that in application for life insurance, cholesterol level of 200 is accepted, while a level of 201 is not. This precision may, in fact, lead to antiselection, as two individuals on the different sides of the dividing line end up being classified differently, even if they are very similar in their basic risk characteristics. Ostaszewski [6] reiterates that argument, and points out that it may be more appropriate not to use preconceived standards, but rather group risks into clusters using crisp, or fuzzy clustering methods, and only then classify them.

In many situations, we do know in advance what characteristics a preferred risk possesses. Any applicant can be compared, in terms of measurements featured in the characteristics, to the "ideal" preferred risk, and then a membership degree can be assigned to each measurement. This produces a feature vector of fuzzy measurements describing the individual. By measuring the fuzziness of that individual as a preferred risk, we can determine a new classification.

0-8186-2812-X/92 $03.00 © 1992 IEEE

Measures of fuzziness indicate the degree of fuzziness of a fuzzy set. De Luca and Termini [2] introduced the classical *entropy measure*, while Yager [9] gave an alternative based on the distance between a fuzzy set and its complement. Ebanks [3] discusses axiomatic properties which a measure of fuzziness should possess, and shows that for a finite universe U = $\{x_1, x_2, \ldots, x_n\}$, and its fuzzy subset $\tilde{E}$ with the membership function μ: $U \rightarrow [0, 1]$, the unique measure satisfying the desired axioms is defined by

$$M(\tilde{E}) = \sum_{i=1}^{n} \mu(x_i)\,(1 - \mu(x_i))$$

- we will call this measure the *axiomatic product measure*, it has been known in the field of fuzzy set theory (Zimmermann [11]).

Let us illustrate the use of measures of fuzziness to classification with the use of a simple example. Among the desired characteristics of a preferred risk for life insurance underwriting we could list: (1) total level of cholesterol in blood of less than 200 mg/dl; (2) systolic blood pressure (in mg of Hg) of less than 130; (3) ratio of body weight to the recommended weight, as a function of height and build of approximately 1; and (4) not smoking. For a given individual, we can determine a degree (a number between 0 and 1) to which that individual satisfies requirements (1) through (4), thus describing each individual with a vector (s_1, s_2, s_3, s_4), and then the overall degree of membership in the preferred risk category (e.g., by using the fuzzy intersection operator as Lemaire [5], or fuzzy inference, as Ostaszewski [6]).

This results in a transformation of the set of individuals considered for insurance into elements of the universe for a fuzzy preferred risk, each of those individuals equipped with his degree of membership in the "ideal" preferred risk category. It is clear that the main purpose of classification is to separate the high risk group from the low risk. We want, therefore, to identify a subset of "good risks" of the universe whose fuzziness is minimized.

De Luca and Termini's [2] entropy measure of fuzziness turns out to be inappropriate for direct application in this area, as it is always minimized for a one-element preferred risk set. The Yager's distance measure produces very satisfactory results, by grouping those individuals who are relatively "close" to the "ideal" risk in one cluster, and the rest in the other. On the other hand, if α–cuts are used to arrive at a crisp risk set (this method is used by Lemaire [5]), instead of minimizing fuzziness, the entropy measure can be used to compare the applicability of various α–cuts. Ebanks' axiomatic measure of fuzziness, although influenced by the number of objects is a set considered, is very appropriate for evaluating fuzziness of fuzzy preferred risk, due to its generalized additivity property, and the fact that it is a valuation on $[0, 1]^U$ (see [3]).

Conclusions

Our work indicates that measures of fuzziness can be used as a tool in classification of risk considered for underwriting. The distance measure of Yager can be utilized to produce a fuzzy preferred risk whose fuzziness is minimized, while the entropy measure and axiomatic measure of fuzziness of Ebanks [3] are tools to evaluate a fuzzy set of preferred risks.

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