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Errata

Effective July 5, 2013, only the latest edition of this manual will have its errata updated. You can find the errata for all latest editions of my books at: http://smartURL.it/errata

Posted April 5, 2013
The second to last formula in the solution of Problem 12 in Practice Examination 5 should be:
\[
\Pr(X > 1) = \int_1^2 \frac{3}{4} x (2 - x) \, dx = \left[ \frac{3}{4} x^2 - \frac{1}{4} x^3 \right]_1^2 = (3 - 2) - \left( \frac{3}{4} - \frac{1}{4} \right) = \frac{1}{2}.
\]
The calculation shown was correct but there was a typo in limits of integral calculation.

Posted April 2, 2013
The last formula in the solution of Problem 10 in Practice Examination 4 should be
\[
E(Y) = \int_{0.6}^{\infty} x \cdot \left( \frac{2.5 \cdot 0.6^{2.5}}{x^{3.5}} \right) \, dx + \int_{2}^{\infty} 2 \cdot \left( \frac{2.5 \cdot 0.6^{2.5}}{x^{3.5}} \right) \, dx + \int_{2}^{\infty} 5 \cdot 0.6^{2.5} \, dx =
\]
\[
= -\left. \frac{2.5 \cdot 0.6^{2.5}}{1.5 x^{1.5}} \right|_{x=0.6}^{x=2} - \left. \frac{2 \cdot 0.6^{2.5}}{x^{2.5}} \right|_{x=2}^{x=\infty} = -\frac{2.5 \cdot 0.6^{2.5}}{1.5 \cdot 2^{1.5}} + \frac{2.5 \cdot 0.6^{2.5}}{1.5} - 0 + \frac{0.6^{2.5}}{2^{1.5}} \approx 0.934273.
\]
The calculation shown was correct, but a factor in the numerator of the last fraction was mistyped as 2.5, when it should have been 2.

Posted March 6, 2013
In the solution of Problem 6 in Practice Examination 2, the expression rectangle \([0, 2] \times [0, 3]\), should be rectangle \([0, 1] \times [0, 2]\).
This expression appears twice in the solution.

Posted March 10, 2011
The second sentence of the solution of Problem 10 in Practice Examination 1 should be:
As the policy has a deductible of 1 (thousand), the claim payment is
When there is no damage, with probability 0.94,
\[
Y = \begin{cases} 
0, & \text{when } 0 < X < 15, \text{ with probability 0.04}, \\
\max(0, X - 1), & \text{in the case of total loss, with probability 0.02}.
\end{cases}
\]

Posted July 3, 2010
In the solution of Problem 1, Practice Examination 5, at the end of the first part of the fourth sentence of the solution, 5/6 is a typo, it should be 5/36, as used in the formula for \( \Pr(Y = 6) \).

Posted January 5, 2010
In the description of the gamma distribution in Section 2, the condition for the range of its MGF should be \( t < \beta \), not \( 0 < t < \beta \).

Posted June 18, 2009
The last formula in the solution of Problem 19 of Practice Examination 5 should be
\[
\Pr(X \geq 10) \leq \frac{1}{4^2} = \frac{1}{16}
\]
instead of \( \Pr(X \geq 10) < \frac{1}{4^2} = \frac{1}{16} \).

Posted April 6, 2009
In the text of Problem 7 in Practice Examination 3, the word “whwther” should be “whether”.

Posted March 2, 2009
The properties of the cumulant moment-generating function should be:
The cumulant generating function has the following properties:
\[
\psi_X(0) = 0,
\]
\[
\frac{d}{dt} \ln E(e^{\lambda X}) \bigg|_{\lambda=0} = \frac{E(X e^{\lambda X})}{E(e^{\lambda X})} \bigg|_{\lambda=0} = E(X),
\]
\[
\frac{d^2}{dt^2} \psi_X(t) \bigg|_{t=0} = \frac{d}{dt} \frac{E(X e^{\lambda X})}{E(e^{\lambda X})} \bigg|_{\lambda=0} = \frac{E(X^2 e^{\lambda X})E(e^{\lambda X}) - E(X e^{\lambda X})^2}{(E(e^{\lambda X}))^2} \bigg|_{\lambda=0} = \text{Var}(X),
\]
\[
\frac{d^3}{dt^3} \psi_X(t) \bigg|_{t=0} = E \left( (X - E(X))^3 \right),
\]
but for \( k > 3 \),
\[ \frac{d^k}{dt^k} \psi_X(t) \bigg|_{t=0} = \psi_X^{(k)}(0) < E \left( (X - E(X))^k \right). \]

Also, if \( X \) and \( Y \) are independent (we will discuss this concept later),
\[ \psi_{aX+b}(t) = \psi_X(at) + bt, \]
and
\[ \psi_{X+Y}(t) = \psi_X(t) + \psi_Y(t). \]

**Posted September 1, 2008**
In the solution of Problem 16 in Practice Examination 2, the left-hand side of the second formula should be \( E(X^2) \), not \( E(X) \).

**Posted July 23, 2008**
In the solution of Problem 5 in Practice Examination 3 the expression \( \text{Var}(X_2) = 40,000 \) should be \( \text{Var}(X_2) = 250,000 \).

**Posted February 7, 2008**
The discussion of the lack of memory property of the geometric distribution should have the formula
\[ \Pr(X = n + k | X \geq n) = \Pr(X = k) \]
corrected to:
\[ \Pr(X = n + k | X > n) = \Pr(X = k). \]

**Posted November 13, 2007**
In the solution of Problem 26 of Practice Examination 5, the sentence:
Of the five numbers, 1 can never be the median.
should be
Of the five numbers, neither 1 nor 5 can ever be the median.

**Posted October 5, 2007**
In the solution of Problem No. 1 in Practice Examination No. 3, the random variable \( Y \) should refer to class B.

**Posted September 13, 2007**
The beginning of the third sentence in the solution of Problem No. 21 in Practice Examination No. 2 should be
The mean error of the 48 rounded ages,
instead of
The mean of the 48 rounded ages.

**Posted May 9, 2007**

In the solution of Problem No. 15, Practice Examination No. 3, the derivative is missing a minus, and it should

\[
\frac{dX}{dY} = -\frac{1}{3}(8 - Y)^{-\frac{2}{3}}.
\]

The rest of the solution is unaffected.

**Posted May 8, 2007**

Problem No. 24 in Practice Examination No. 5 should be (it had a typo in the list of possible values of x).

**May 1992 Course 110 Examination, Problem No. 35**

Ten percent of all new businesses fail within the first year. The records of new businesses are examined until a business that failed within the first year is found. Let X be the total number of businesses examined prior to finding a business that failed within the first year. What is the probability function for X?

A. 0.1 \cdot 0.9^x, for x = 0, 1, 2, 3, ...
B. 0.9 \cdot 0.1^x, for x = 0, 1, 2, 3, ...
C. 0.1x \cdot 0.9^x, for x = 1, 2, 3, ...
D. 0.9x \cdot 0.1^x, for x = 1, 2, 3, ...
E. 0.1 \cdot (x - 1 \cdot 0.9^x), for x = 2, 3, 4, ...

**Solution.**

Let a failure of a business be a success in a Bernoulli Trial, and a success of a business be a failure in the same Bernoulli Trial. Then X has the geometric distribution with \( p = 0.1 \), and therefore \( f_X(x) = 0.1 \cdot 0.9^x \) for \( x = 0, 1, 2, 3, \ldots \).

Answer A.

**Posted May 5, 2007**

Problem No. 10 in Practice Examination No. 3 should be (it had a typo in one of the integrals and I decided to reproduce the whole problem to provide a better explanation):

**Sample Course 1 Examination, Problem No. 35**

Suppose the remaining lifetimes of a husband and a wife are independent and uniformly distributed on the interval (0, 40). An insurance company offers two products to married couples:

- One which pays when the husband dies; and
- One which pays when both the husband and wife have died.

Calculate the covariance of the two payment times.
Solution.
Let $H$ be the random time to death of the husband, $W$ be the time to death of the wife, and $X$ be the time to the second death of the two. Clearly, $X = \max(H, W)$. We have

$$f_H(h) = f_W(w) = \frac{1}{40}$$

for $0 \leq h \leq 40$, and $0 \leq w \leq 40$. Thus $\mathbb{E}(H) = \mathbb{E}(W) = 20$. Furthermore,

$$F_X(x) = \Pr(X \leq x) = \Pr(\max(H, W) \leq x) = \Pr(\{H \leq x\} \cap \{W \leq x\}) = \Pr(H \leq x) \cdot \Pr(W \leq x) = \frac{x}{40} \cdot \frac{x}{40} = \frac{x^2}{1600}.$$  

This implies that $s_X(x) = 1 - \frac{x^2}{1600}$ for $0 \leq x \leq 40$, and

$$\mathbb{E}(X) = \int_0^{40} \left(1 - \frac{x^2}{1600}\right) dx = 40 - \frac{40^3}{4800} = \frac{120}{3} - \frac{40}{3} = \frac{80}{3}.$$  

In order to find covariance, we also need to find $\mathbb{E}(XH) = \mathbb{E}(\max(H, W))$. We separate the double integral into two parts: one based on the region where the wife lives longer and one based on the region where the husband lives longer, as illustrated in the graph below.

$$E(\max(H, W)) = \int \int \max(h, w) \cdot f_H(h) \cdot f_W(w) \cdot dw \, dh =$$

$$= \int_{h=0}^{40} \left(\int_{w=0}^{h} h^2 \cdot \frac{1}{40} \cdot \frac{1}{40} \, dw\right) dh + \int_{h=0}^{40} \left(\int_{w=0}^{h} hw \cdot \frac{1}{40} \cdot \frac{1}{40} \, dw\right) dh =$$

$$= \frac{40}{1600} \int_{h=0}^{40} \left(\frac{h^3}{3200}\right) dh + \frac{40}{3200} \int_{h=0}^{40} \left(\frac{h^4}{1600}\right) dh = \frac{40}{1600} \int_{h=0}^{40} h^3 \, dh + \frac{40}{3200} \int_{h=0}^{40} (1600h - h^3) \, dh =$$

$$= \frac{40}{2} \int_{h=0}^{40} h^3 \, dh = \left(\frac{1}{4} h^4 + \frac{1}{12800} h^4\right)_{h=0}^{h=40} = \frac{1}{4} \cdot 40^2 + \frac{1}{12800} \cdot 40^4 = 600.$$
Finally,
\[
\text{Cov}(X, H) = E(XH) - E(X)E(H) = 600 - 20 \cdot \frac{80}{3} = 66\frac{2}{3}.
\]

Answer C.

Posted April 29, 2007
The second formula to the last in the solution of Problem No. 6 in Practice Examination No. 3 should be:

\[
E(X^2) = \frac{d^2 M(t)}{dt^2} \bigg|_{t=0} = \left. \frac{d}{dt} 3e^t \left( \frac{2 + e^t}{3} \right)^8 \right|_{t=0} = \left. 3e^t \left( \frac{2 + e^t}{3} \right)^8 + 8e^t \left( \frac{2 + e^t}{3} \right)^7 \cdot e^t \right|_{t=0} = 11.
\]

instead of

\[
E(X^2) = \frac{d^2 M(t)}{dt^2} \bigg|_{t=0} = \left. \frac{d}{dt} 3e^t \left( \frac{2 + e^t}{3} \right)^8 \right|_{t=0} = \left. 3e^t \left( \frac{2 + e^t}{3} \right)^8 + 8e^t \left( \frac{2 + e^t}{3} \right)^7 \cdot e^t \right|_{t=0} = 11.
\]

Posted April 27, 2007
In the solution of Problem No. 9 in Practice Examination No. 3, the expression \( \max(X, 1) \) should be \( \min(X, 1) \).

Posted April 20, 2007
Problem No. 16 in Practice Examination No. 1 should be:

May 2001 Course 1 Examination, Problem No. 26, also Study Note P-09-05, Problem No. 109

A company offers earthquake insurance. Annual premiums are modeled by an exponential random variable with mean 2. Annual claims are modeled by an exponential random variable with mean 1. Premiums and claims are independent. Let \( X \) denote the ratio of claims to premiums. What is the density function of \( X \)?

A. \( \frac{1}{2x+1} \)  
B. \( \frac{2}{(2x+1)^2} \)  
C. \( e^{-x} \)  
D. \( 2e^{-2x} \)  
E. \( xe^{-x} \)

Solution.
Let \( U \) be the annual claims and let \( V \) be the annual premiums (also random), and let \( f_{U,V}(u,v) \) be the joint density of them. Also, let \( f_X \) be the density of \( X \) and let \( F_x \) be its cumulative distribution function. We are given that \( U \) and \( V \) are independent, so that

\[
f_{U,V}(u,v) = e^{-u} \cdot \frac{1}{2} e^{-\frac{v}{2}} = \frac{1}{2} e^{-u} e^{-\frac{v}{2}} \text{ for } 0 < u < \infty, 0 < v < \infty.
\]

Also, noting the graph below,

\[
u = vx \text{ or } v = u/x
\]
we have:

\[ F_X(x) = \Pr(X \leq x) = \Pr\left(\frac{U}{V} \leq x\right) = \Pr(U \leq Vx) = \int_{0}^{\infty} \int_{0}^{\infty} f_{U,V}(u,v) \, dv \, du = \]

\[ = \int_{0}^{\infty} \int_{0}^{\infty} \frac{1}{2} e^{-u} e^{-\frac{v}{2}} \, dv \, du = \int_{0}^{\infty} \int_{0}^{\infty} \left(-\frac{1}{2} e^{-u} e^{-\frac{v}{2}} + \frac{1}{2} e^{-\frac{v}{2}}\right) \, dv \, du = \]

\[ = \int_{0}^{\infty} \left(-\frac{1}{2} e^{-\frac{v}{2}(x^2 + 1)} + \frac{1}{2} e^{-\frac{v}{2}}\right) \, dv = \left(\frac{1}{2x+1} e^{-\frac{v}{2}(x^2 + 1)} - e^{-\frac{v}{2}}\right) \bigg|_{0}^{\infty} = -\frac{1}{2x+1} + 1. \]

Finally,

\[ f_X(x) = F'_X(x) = \frac{2}{(2x+1)^2}. \]

Answer B. This problem can also be done with the use of bivariate transformation. We will now give an alternative solution using that approach. Consider the following transformation \( X = \frac{U}{V}, Y = V \). Then the inverse transformation is \( U = XY, V = Y \). We know that

\[ f_{U,V}(u,v) = \frac{1}{2} e^{-u} e^{-\frac{v}{2}} \]

for \( u > 0, v > 0 \). It follows that

\[ f_{X,Y}(x,y) = f_{U,V}(u(x,y),v(x,y)) \left| \frac{\partial(u,v)}{\partial(x,y)} \right| = \frac{1}{2} e^{-xy} e^{-\frac{y}{2}} \left| \det \begin{bmatrix} y & x \\ 0 & 1 \end{bmatrix} \right| = \frac{1}{2} ye^{-xy} e^{-\frac{y}{2}} \]

for \( xy > 0 \) and \( y > 0 \), or just \( x > 0 \) and \( y > 0 \). Therefore,
Answer B, again. The second approach is probably more complicated, but it is a good exercise in the use of multivariate transformations.

**Posted January 20, 2007**
**Exercise 2.8 should read as follows:**
**November 2001 Course 1 Examination, Problem No. 11**
A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is \( \frac{3}{5} \). The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months. Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

A. 0.01   B. 0.12   C. 0.23   D. 0.29   E. 0.41

Solution.
Consider a Bernoulli Trial with success defined as a month with an accident, and a month with no accident being a failure. Then the probability of success is \( \frac{3}{5} \). Now consider a negative binomial random variable, which counts the number of failures (months with no accident) until 4 successes (months with accidents), call it \( X \). The problem asks us to find \( \Pr(X \geq 4) \).

\[
f_{X}(x) = \int_{0}^{\infty} \frac{1}{2} ye^{-\frac{y}{2}} e^{-\frac{x+y}{2}} dy = \int_{0}^{\infty} \frac{1}{2} ye^{-\left(\frac{x+y}{2}\right)} dy = \frac{w = \frac{1}{2} y}{\int_{0}^{\infty} \frac{1}{2} \left(1 - e^{-\left(\frac{x+y}{2}\right)}\right) dy = dz = e^{-\left(\frac{x+y}{2}\right)} dy}
\]

INTEGRATION BY PARTS

\[
= \left[-\frac{1}{2} y \cdot \frac{1}{\left(x + \frac{1}{2}\right)} e^{-\left(\frac{x+y}{2}\right)}\right]_{y=0}^{\infty} - \int_{0}^{\infty} \frac{1}{2} \left(-\frac{1}{\left(x + \frac{1}{2}\right)}\right) e^{-\left(\frac{x+y}{2}\right)} dy = 0 + \int_{0}^{\infty} \frac{1}{2x+1} e^{-\left(\frac{x+y}{2}\right)} dy = \left[\frac{1}{2x+1} \left(1 - e^{-\left(\frac{x+y}{2}\right)}\right)\right]_{y=0}^{\infty} = \frac{2}{(2x+1)^2}.
\]
\[
\Pr(X \geq 4) = 1 - \Pr(X = 0) - \Pr(X = 1) - \Pr(X = 2) - \Pr(X = 3) = \\
= 1 - \left( \binom{3}{0} \cdot \left( \frac{3}{5} \right)^4 \right) - \left( \binom{4}{1} \cdot \left( \frac{3}{5} \right)^4 \cdot \frac{2}{5} \right) - \left( \binom{5}{2} \cdot \left( \frac{3}{5} \right)^4 \cdot \frac{2}{5} \right) - \left( \binom{6}{3} \cdot \left( \frac{3}{5} \right)^4 \cdot \frac{2}{5} \right)^3 \\
\approx 0.289792.
\]

Answer D.

**Posted December 31, 2006**

**In Section 2, the general definition of a percentile should be**

the 100-\(p\)-th percentile of the distribution of \(X\) is the number \(x_p\) which satisfies both of the following inequalities: \(\Pr(X \leq x_p) \geq p\) and \(\Pr(X \geq x_p) \geq 1 - p\).

**It was mistyped as**

the 100-\(p\)-th percentile of the distribution of \(X\) is the number \(x_p\) which satisfies both of the following inequalities: \(\Pr(X \leq x_p) \geq p\) and \(\Pr(X \geq x_p) \leq 1 - p\).

**Posted December 28, 2006**

**On page 3 in the bottom paragraph, the words:**
is also defined as the set that consists of all elementary events that belong to any one of them.

**should be**
is also defined as the set that consists of all elementary events that belong to any one of them.

The word “set” was mistyped as “sent.”

**Posted November 25, 2006**

**The third equation from the bottom in the solution of Problem No. 10 in Practice Examination No. 2 should be**

\[
\Pr(A \cap E_{99}) = \Pr(A|E_{99}) \cdot \Pr(E_{99}) = 0.03 \cdot 0.20 = 0.006.
\]

**Previously it had a typo:**

\[
\Pr(A \cap E_{99}) = \Pr(A|E_{99}) \cdot \Pr(E_{99}) = 0.03 \cdot 0.20 = 0.0036.
\]

**Posted August 3, 2006**

**In the solution of Problem No. 4 in Practice Examination No. 1, the formula**

\[
f_Y(y) = k_2 \cdot y,
\]

**should be**

\[
f_Y(y) = k_2 \cdot 1.
\]

**Posted July 3, 2006**
The relationship between the survival function and the hazard rate, stated just after the definition of the hazard rate, should be

\[ s_X(x) = e^{-\int_{-\infty}^{x} \lambda(u) \, du} \]

instead of

\[ s_X(x) = e^{- \int_{0}^{x} \lambda(u) \, du} \].

The second equation applies in the case when the random variable \( X \) is nonnegative almost surely.

Posted July 3, 2006
The condition concerning a continuous probability density function in its first definition, in Section 1, should be

\[ f_X(x) \geq 0 \text{ for every } x \in \mathbb{R} \]

instead of

\[ 0 \leq f_X(x) \leq 1 \text{ for every } x \in \mathbb{R}. \]

Posted May 16, 2006
The solution of Problem No. 13 in Practice Examination No. 3 should be (some of the exponents contained typos)

Solution.
Let \( X \) be the random number of passengers that show for a flight. We want to find

\[ \Pr(X = 31) + \Pr(X = 32). \]

We can treat each passenger arrival as a Bernoulli Trial, and then it is clear that \( X \) has binomial distribution with \( n = 32, p = 0.90 \). Therefore,

\[ \Pr(X = x) = \binom{32}{x} \cdot 0.90^x \cdot 0.10^{32-x}. \]

The probability desired is:

\[ \Pr(X = 31) + \Pr(X = 32) = \binom{32}{31} \cdot 0.90^{31} \cdot 0.10^1 + \binom{32}{32} \cdot 0.90^{32} \cdot 0.10^0 \approx \]

\[ \approx 0.12208654 + 0.03433684 = 0.15642337. \]

Answer E.

Posted March 1, 2006
The formula for the variance of a linear combination of random variables should be:
\[
\text{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n) = \\
= [a_1 \ a_2 \ \ldots \ \ a_n] \cdot \left[ \text{Cov}(X_i, X_j) \right]_{i,j=1,2,\ldots,n} \cdot [a_1 \ a_2 \ \ldots \ \ a_n]^T = \\
= [a_1 \ a_2 \ \ldots \ \ a_n] \cdot \left[ \text{Cov}(X_i, X_j) \right]_{i,j=1,2,\ldots,n} \cdot \\
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n \\
\end{bmatrix} = \\
= \sum_{i=1}^{n} a_i^2 \text{Var}(X_i) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} 2a_i a_j \cdot \text{Cov}(X_i, X_j). \\
\]

The only incorrect piece was in the last double sum, where \( a_i a_2 \) appeared instead of \( a_i a_j \).

**Posted February 11, 2006**

The solution of Problem No. 18 in Practice Examination No. 4 should be:

Let \( X \) be the claim for employee who incurred a loss in excess of 2000, and \( Y \) be the claim for the other employee. The joint distribution of \( X \) and \( Y \), given that each loss occurs, is uniform on \([1000, 5000]^2\). This means that we can calculate all probabilities by comparing areas. The probability \( X + Y > 8000 \), i.e., total losses exceed reimbursement, given that \( X > 2000 \) is the ratio of the area of the triangle in the right upper corner of the square \([1000, 5000]^2\) in the figure below to the area of the rectangle \([2000, 5000] \times [1000, 5000]\), and that ratio is \( \frac{1}{2} \cdot 2000 \cdot 2000 = \frac{1}{6} \).

![Diagram](attachment:diagram.png)
However, this is conditional on the occurrence of the loss of the employee whose loss is $Y$ (and not conditional on the occurrence of the loss for the employee whose loss is $X$, because the event considered is already conditional on $X > 2000$, so the loss has occurred). The probability of the loss occurring is 40%. Therefore the probability sought is

\[ 40\% \cdot \frac{1}{6} = \frac{2}{5} \cdot \frac{1}{6} = \frac{1}{15}. \]

Answer B.

**Posted January 31, 2006**

In the solution of Problem No. 21 (May 2000 Course 1 Examination, Problem No. 19) in Practice Examination No. 2, the fourth sentence should begin

The mean of the 48 rounded ages, \( \bar{E} = \frac{1}{48} \sum_{i=1}^{48} E_i \).

**Posted January 26, 2006**

In the solution of Problem No. 15 (May 2000 Course 1 Examination, Problem No. 7) in Practice Examination No. 2, the survival function should be written as:

\[ s_X(x) = (1 + x)^{-3}. \]

**Posted January 18, 2006**

In the description of the geometric distribution:

\[ M_X(t) = \frac{pe^t}{1 - qe^t}. \]

Some texts use a different version of this distribution, which only counts failures until the first success. That’s just too pessimistic for this author. The two distributions differ by one. For this other form of random variable (similar in its design to the negative binomial distribution, just below) \( Y = X - 1 \), and \( E(Y) = \frac{q}{p} \), \( \text{Var}(X) = \frac{q}{p^2} \),

\[ M_X(t) = \frac{p}{1 - qe^t}. \]

should be

\[ M_X(t) = \frac{pe^t}{1 - qe^t}. \]

Some texts use a different version of this distribution, which only counts failures until the first success. That’s just too pessimistic for this author. The two distributions differ by one. For this other form of random variable (similar in its design to the negative binomial distribution, just below) \( Y = X - 1 \), and \( E(Y) = \frac{q}{p} \), \( \text{Var}(Y) = \frac{q}{p^2} \),

\[ M_Y(t) = \frac{p}{1 - qe^t}. \]
Posted September 8, 2005
In the solution of Exercise 2.2, the first sentence should be:
It is important to realize that there is a point-mass at 1 -- this can be seen by analyzing the limit of CDF at 1 from the left (equal to 0) and the right-hand side limit of CDF at 1 (equal to 0.5).

Posted August 21, 2005
In Exercise 3.11, the second to last formula should be:
\[ f_X(2) = \int_1^\infty \frac{1}{2}y^{-3}dy = -\frac{1}{4}y^{-2} \bigg|_1^\infty = \frac{1}{4}. \]
It was missing \( y^{-2} \).

Posted August 20, 2005
Exercise 3.15, November 2001 Course 1 Examination, Problem No. 40, in the solution, first two displayed formulas should be:
\[ E(X+Y) = E(X) + E(Y) = 50 + 20 = 70, \]
\[ \text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X,Y) = 50 + 30 + 20 = 100. \]
The problem with existing formulas was that \( Y \) was improperly replaced by \( X \).

Posted August 8, 2005
In Problem No. 7, Practice Examination No. 4, the five answer choices should be:
A. \( \frac{6}{125,000} \int_0^{20} \int_0^{20} (50-x-y)dydx \)
B. \( \frac{6}{125,000} \int_0^{20} \int_0^{20} (50-x-y)dydx \)
C. \( \frac{6}{125,000} \int_0^{20} \int_0^{50-x-y} (50-x-y)dydx \)
D. \( \frac{6}{125,000} \int_0^{20} \int_0^{50-x-y} (50-x-y)dydx \)
E. \( \frac{6}{125,000} \int_0^{20} \int_0^{50-x-y} (50-x-y)dydx \)

Posted August 5, 2005
The solution of Problem No. 13, Practice Examination No. 5 should be:
Note the region where the density is positive and the region describing the probabilities we are calculating in the figure below.

Therefore,

\[ F_w(w) = \Pr(W \leq w) = \Pr(XY \leq w) = \Pr(Y \leq \frac{w}{X}) = \]

\[ = \sqrt{w} \left( \int_0^w 8xydx \right)dy + \int_0^{\sqrt{w}} \left( \int_0^w 8xydx \right)dy = \int_0^{\sqrt{w}} \left( 4x^2y \right)_{x=0}^ydy + \int_0^{\sqrt{w}} \left( 4x^2y \right)_{x=0}^{\frac{w}{y}}dy = \]

\[ = \sqrt{w} \left( \int_0^w 4y^3dy + \int_0^{\sqrt{w}} \frac{4w^2}{y}dy \right) = \left( y^4 \right)_{y=0}^{\sqrt{w}} + \left( 4w^2 \ln y \right)_{y=1}^{\sqrt{w}} = \]

\[ = \left( w^2 - 0 \right) + 4w^2 \left( \ln 1 - \ln \sqrt{w} \right) = w^2 - 4w^2 \ln \sqrt{w}. \]

Hence,

\[ f_w(w) = F'_w(w) = 2w - 8w \ln \sqrt{w} - 4w^2 \frac{1}{\sqrt{w}} \cdot \frac{1}{2\sqrt{w}} = \]

\[ = 2w - 8w \ln \sqrt{w} - 2w = -8w \ln \sqrt{w}. \]

Answer A.

**Posted June 12, 2005**

In Section 1, the statement of the Binomial Theorem should be:

\[ (a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}, \]

i.e., the sum should start from \( k = 0 \), not \( k = 1 \).

**Posted May 20, 2005**

On the second page of Section 4: Risk and Insurance, page 77 of the manual, the formula for the variance should be:
\[ \text{Var}(X) = E(X^2) - (E(X))^2 = q \cdot E(B^2) - (q \cdot E(B))^2 = q \cdot E(B^2) - q \cdot (1 - p) \cdot (E(B))^2 = \\
q \cdot \left( E(B^2) - (E(B))^2 \right) + pq \cdot (E(B))^2 = q \cdot \text{Var}(B) + pq \cdot (E(B))^2. \]

\[ \text{i.e., } +pq \cdot (E(B))^2 \text{ instead of } -pq \cdot (E(B))^2 \]

**Posted May 18, 2005**

**In Exercise 2.13, the five answer choices should be:**

A. \(10y^{0.8}e^{-8y^{0.2}}\)

B. \(8y^{-0.2}e^{-10y^{-0.8}}\)

C. \(8y^{-0.2}e^{-(0.1y)^{2.25}}\)

D. \((0.1y)^{1.25}e^{-(0.1y)^{2.25}}\)

E. \(0.125 \cdot (0.1y)^{0.25}e^{-(0.1y)^{2.25}}\)

(parentheses were missing in answers D and E)

**Posted May 18, 2005**

**The last line of the solution of Problem No. 11, Practice Examination No. 3, should be:**

\[ E\left( \max(Y + Z, 2) - 2|X = 0 \right) = 1 \cdot \left( \frac{4}{27} + \frac{5}{27} \right) + 2 \cdot \frac{6}{27} = \frac{7}{9}. \]

(2 is multiplied by \(\frac{6}{27}\), not by \(\frac{5}{27}\)).

**Posted May 18, 2005**

**The last line of the solution of Problem No. 26, Practice Examination No. 3, should be:**

\[ E\left( X^2 \right) = \frac{d^2}{dt^2} M_X(t) \bigg|_{t=0} = \frac{d^2}{dt^2} e^{3t^2} \bigg|_{t=0} = \frac{d}{dt} \left( (3 + 2t) e^{3t^2} \right) \bigg|_{t=0} = \\
\left( 2e^{3t^2} + (3 + 2t)^2 e^{3t^2} \right) \bigg|_{t=0} = 2 + 9 = 11. \]

(the superscript 2 was missing in the symbol of the second derivative).
Post May 14, 2005
In Problem No. 25, Practice Examination No. 5, in the solution, this formula
\[
\bar{Y} - \bar{X} = \frac{Y_1 - X_1}{100} + \frac{Y_2 - X_2}{100} + \ldots + \frac{Y_{100} - X_{100}}{100},
\]
should replace
\[
\bar{Y} - \bar{X} = \frac{X_1 - Y_1}{100} + \frac{X_2 - Y_2}{100} + \ldots + \frac{X_{100} - Y_{100}}{100}.
\]

Post May 14, 2005
In Problem No. 9, Practice Examination No. 5, the solution is correct but the answer choice should be E.

Post May 14, 2005
In Problem No. 14, Practice Examination No. 4, the solution is correct but the answer choice should be C.

Post May 14, 2005
The density of the log-normal distribution should be:
\[
f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln x - \mu}{\sigma} \right)^2},
\]
(it was missing the one-half in the numerator).

Post April 28, 2005
On page 10, in the definition of independent sets, subscripts contain typos, they should be:
In general, we say that events $A_1, A_2, \ldots, A_n$ are mutually independent if, for any finite collection of them $A_{i_1}, A_{i_2}, \ldots, A_{i_k}$, where $1 \leq i_1 < i_2 < \ldots < i_k \leq n$, we have
\[
\Pr\left( A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k} \right) = \Pr\left( A_{i_1} \right) \cdot \Pr\left( A_{i_2} \right) \cdot \ldots \cdot \Pr\left( A_{i_k} \right).
\]
Posted April 28, 2005
On page 21, the density introduced there should be:

\[
f_X(x) = \begin{cases} 
0, & x < 0 \text{ or } x > 1, \\
0.10, & x = 0, \\
x, & 0 < x < 1, \\
0.40, & x = 1.
\end{cases}
\]

Posted April 28, 2005
On page 23, instead of

\[
E(X) = 1 \cdot s_X(0) + 1 \cdot s_X(1) + 1 \cdot s_X(1) + \ldots = \sum_{n=0}^{\infty} \Pr(X > n) = \sum_{n=1}^{\infty} \Pr(X \geq n).
\]

the formula should be:

\[
E(X) = 1 \cdot s_X(0) + 1 \cdot s_X(2) + 1 \cdot s_X(3) + \ldots = \sum_{n=0}^{\infty} \Pr(X > n) = \sum_{n=1}^{\infty} \Pr(X \geq n).
\]

Posted April 28, 2005
On page 25, the first formula on this page should be:

\[
E(X^n) = \int_0^{+\infty} x^n \cdot \lambda e^{-\lambda x} \, dx = \int_0^{+\infty} x^n \cdot d(-e^{-\lambda x}) = x^n \cdot \left( -e^{-\lambda x} \right) \bigg|_{x=0}^{x=+\infty} + \int_0^{+\infty} nx^{n-1} \cdot e^{-\lambda x} \, dx = \\
= 0 + \frac{n}{\lambda} \int_0^{+\infty} x^{n-1} \cdot e^{-\lambda x} \, dx = \frac{n}{\lambda} E(X^{n-1}).
\]

instead of:

\[
E(X^n) = \int_0^{+\infty} x^n \cdot \lambda e^{-\lambda x} \, dx = \int_0^{+\infty} x^n \cdot d(-e^{-\lambda x}) \lambda = x^n \cdot \left( -e^{-\lambda x} \right) \bigg|_{x=0}^{x=+\infty} + \int_0^{+\infty} nx^{n-1} \cdot e^{-\lambda x} \, dx = \\
= 0 + \frac{n}{\lambda} \int_0^{+\infty} x^{n-1} \cdot e^{-\lambda x} \, dx = \frac{n}{\lambda} E(X^{n-1}).
\]

Posted April 28, 2005
On page 41, derivation of the density of a transformation of the random variable should be corrected as follows:
\[ f_y(y) = F'_y(y) = \begin{cases} \frac{d}{dy} F_x(\varphi^{-1}(y)), & \text{if } \varphi \text{ is increasing,} \\ \frac{d}{dy} s_x(\varphi^{-1}(y)), & \text{if } \varphi \text{ is decreasing.} \end{cases} \]

\[ = \begin{cases} f_x(\varphi^{-1}(y)) \frac{d}{dy} \varphi^{-1}(y), & \text{if } \varphi \text{ is increasing,} \\ -f_x(\varphi^{-1}(y)) \frac{d}{dy} \varphi^{-1}(y), & \text{if } \varphi \text{ is decreasing.} \end{cases} \]

\[ = \begin{cases} f_x(\varphi^{-1}(y)) \frac{1}{\varphi'(\varphi^{-1}(y))}, & \text{if } \varphi \text{ is increasing,} \\ f_x(\varphi^{-1}(y)) \left( -\frac{1}{\varphi'(\varphi^{-1}(y))} \right), & \text{if } \varphi \text{ is decreasing.} \end{cases} \]

At the end of this derivation, the text should be:

There is a less formal, but more intuitive, way to write all of this. Let us think this way:

\[ Y = \varphi(X) = Y(X), \]

and

\[ X = \varphi^{-1}(Y) = X(Y), \]

\[ \varphi'(X) = \frac{dY}{dX}, \]

\[ (\varphi^{-1})'(Y) = \frac{dX}{dY}. \]

Then

\[ F_y(y) = \begin{cases} F_x(x(y)), & \text{if } \frac{dy}{dx} > 0, \\ s_x(x(y)), & \text{if } \frac{dy}{dx} < 0. \end{cases} \]

\[ f_y(y) = f_x(x(y)) \left| \frac{dx}{dy} \right|. \]

Posted April 28, 2005

On page 52, the formulas for mixed distributions should be:

\[ f_x(x) = \alpha_1 f_{x_1}(x) + \ldots + \alpha_n f_{x_n}(x), \]

\[ F_x(x) = \alpha_1 F_{x_1}(x) + \ldots + \alpha_n F_{x_n}(x), \]

\[ E\left(X^n\right) = \alpha_1 E\left(X_1^n\right) + \ldots + \alpha_n E\left(X_n^n\right). \]
Posted April 28, 2005
On page 25, in the solution of Exercise 2.1, the formula should be:

\[ E(X) = \int_{-2}^{0} x \left( -\frac{x}{10} \right) dx + \int_{0}^{4} x \cdot \frac{x}{10} dx = \frac{-8}{30} + \frac{64}{30} = \frac{56}{30} = \frac{28}{15}. \]

instead of

\[ E(X) = \int_{-2}^{0} x \left( -\frac{x}{10} \right) dx + \int_{0}^{4} x \cdot \frac{x}{10} dx = \frac{-8}{30} + \frac{64}{30} = \frac{56}{30} = \frac{28}{15}. \]

Posted April 28, 2005
On pages 44-45, the last sentence of the solution of the Exercise 2.14 should be
We conclude that we should choose the payment amount \( C \) such that \( 2C = 120 \) or \( C = 60 \).

Posted April 28, 2005
On page 39, the PDF of the gamma distribution should be:

\[ f_X(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{(\alpha-1)!} \quad \text{or} \quad f_X(x) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}. \]

Posted April 28, 2005
On page 65, Answer E to Exercise 3.7 should be: \( e^{t_1 + t_2} \).

Posted April 28, 2005
On page 78, the text should be:
For example, in a policy with a deductible \( d \), we have:

\[ Y = (X - d) | X > d. \]

Posted April 28, 2005
On page 83, in the solution of Exercise 4.7 the variance should be:

\[ \text{Var}(Y) = E(Y^2) - (E(Y))^2 = 434,027.78 - 520.83^2 = 162,763.89. \]

But the standard deviation is correctly calculated as 403. This means that the correct answer is B.

Posted April 28, 2005
On page 94, the third sentence of the solution of Problem No. 2 in Practice Examination 1 should be: 
The shaded region is the portion of the domain over which $X < 0.2$.

Posted April 21, 2005
On page 37, the normal PDF density should be:

$$f_X(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < +\infty,$$

Posted April 10, 2005
On page 39, following the discussion of the gamma distribution, the following should be added

*Chi-square distribution*

A gamma distribution for which $\alpha = \frac{n}{2}$ and $\beta = \frac{1}{2}$ is called the *chi-square distribution* with $n$ degrees of freedom, with $n$ being a positive integer. Its density is

$$f_X(x) = \frac{1}{2^n \Gamma\left(\frac{n}{2}\right)} x^{\frac{n-1}{2}} e^{-\frac{x}{2}}, \text{ for } x > 0,$$

and zero otherwise.

We have

$$E(X) = n, \ Var(X) = 2n, \text{ and } M_X(t) = \left(\frac{1}{1-2t}\right)^{\frac{n}{2}} \text{ for } 0 < t < \frac{1}{2}.$$ 

There are two very important properties of the chi-square distribution that you should remember:

- If $Z_1, Z_2, \ldots, Z_n$ is a random sample (i.e., a set of independent identically distributed random variables) from a standard normal distribution, then $Z_1^2 + Z_2^2 + \ldots + Z_n^2$ has the chi-square distribution with $n$ degrees of freedom.

- If $X$ and $Y$ are independent and have the standard normal distribution then $X^2 + Y^2$ has the chi-square distribution with 2 degrees of freedom, i.e., the gamma distribution with parameters $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$. But any gamma distribution with $\alpha = 1$ is an exponential distribution whose parameter is $\lambda = \beta$ and mean is $\frac{1}{\beta}$. Thus $X^2 + Y^2$ has the exponential distribution with mean 2 (or parameter $\lambda = \frac{1}{2}$).
Posted March 22, 2005
On page 9, in the third paragraph from the bottom, in the following two sentences, the underlined words should be (as they are here): “one”, “of”, instead of “once”, “if”.
This means that two events are independent if occurrence of one of them has no effect on the probability of the other one happening. You must remember that the concept of independence should never be confused with the idea of two events being mutually exclusive.

Posted March 24, 2005
In the solution of Exercise 1.4
We need to find \( \Pr(M^C \cap F^C) \).
should be
We need to find \( \Pr(M^C \cap C^C) \).

Posted March 12, 2005
On page 38, at the end of line 9, the PDF of the exponential distribution should be
\[ f_T(t) = \lambda e^{-\lambda t}, \text{ not } f_T(t) = \frac{1}{\beta} e^{-\beta t}. \]
Similarly, in terms of the mean, the PDF should be
\[ f_T(t) = \frac{1}{\mu} e^{-\frac{t}{\mu}}. \]

Posted March 26, 2005
On page 39, following the discussion of the Pareto distribution, add:
Another form of the Pareto distribution that is often considered has \( x - x_0 \) as the variable (i.e., \( x \) replaces \( x - x_0 \) in formulas above), and for that form of the Pareto distribution,
\[ f_X(x) = \frac{\alpha x_0^\alpha}{(x + x_0)^{\alpha + 1}}, \text{ for } x > 0, \]
\[ E(X) = \frac{x_0}{\alpha - 1}, \]
\[ \text{Var}(X) = \frac{\alpha x_0^2}{(\alpha - 2)(\alpha - 1)^2}, \]
\[ s_X(x) = \frac{x_0^\alpha}{(x + x_0)^{\alpha}}, \text{ for } x > 0. \]
Posted March 1, 2005
On pages 81 and 88, answer E to Problem No. 1 in Practice Examination No. 1 should be
\[
\frac{1}{4} \pi \int_{\frac{2}{3}}^{\frac{5}{3}} x \cos \pi x (1 + \sin \pi x)^3 \, dx.
\]

Posted March 1, 2005
On pages 83 and 93, answer A to Problem No. 9 in Practice Examination No. 1 should be:
\[
\frac{1}{12}.
\]

Posted March 1, 2005
On pages 84 and 96, in Problem No. 14 of Practice Examination No. 1, “flue” should be replaced by “flu.”