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## **The Inflection Point of the Laffer Curve**

### **Abstract**

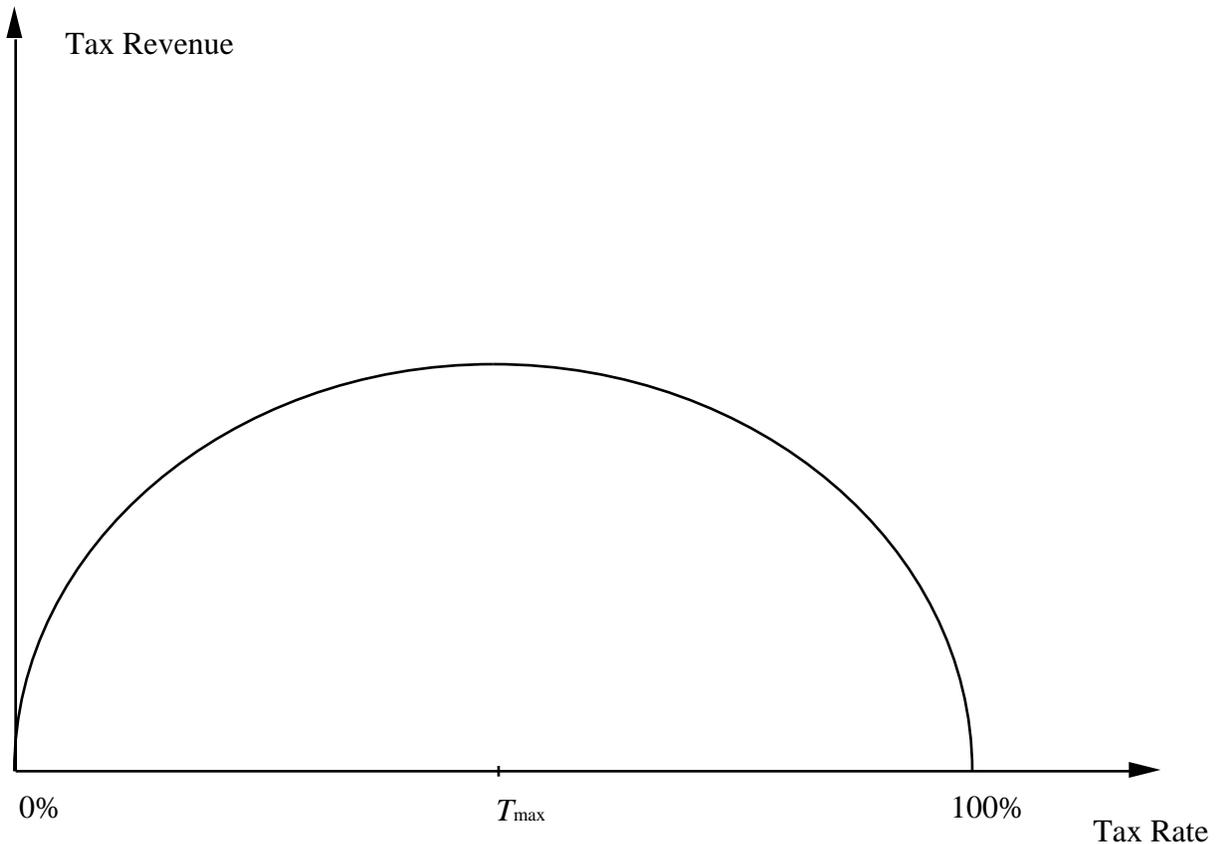
The relationship of the tax revenues to the tax rate is often referred to as the *Laffer Curve*. In this note we investigate the nature of this relationship, and point out that the standard approach to the curve ignores the possibility that its shape may reveal information about the economy, and the crucial portion of such information is conveyed in the inflection point of the curve. We point out that one should expect the Laffer Curve to have an inflection point. Then we discuss all possible choices of effective tax rates, and their effects on tax revenues and national income.

## Introduction

The Laffer Curve, i.e., the curve illustrating the relationship of total tax revenues to the tax rate, is one of the most successful and controversial applications of economic ideas to politics. As the story goes, Arthur Laffer drew his curve representing the graph of the relationship of total tax revenues to the tax rate on a napkin in order to convince Ronald Reagan that it was possible that lower tax rates can result in higher tax revenues. The idea was one of the forces behind the cuts in personal income tax rates in the 1980s (see [2]). It has been hailed and ridiculed, and it remains a politically charged issue.

In this note, we would like to offer some further analysis of the Laffer Curve. We believe that there is more to the story, and that its continuation is quite revealing. Let us start with the basics. Our objective is to better understand the relationship between tax rates and total tax revenue. It is quite obvious that if the tax rate is set at 0%, no revenues will be collected. It is nearly as obvious, although less politically neutral, that if the tax rate is set at 100%, no revenues will be collected (one could argue that some collection could occur for a limited period of time, as this scenario is equivalent to nationalization or other form of expropriation). As the tax rates vary continuously between 0% and 100%, total tax revenues also vary. People adjust their behavior in relation to tax rates, and one cannot simply assume that higher tax rates result in higher revenues. Undoubtedly, there exist numerous examples of situations when higher tax rates did result in higher revenues (the 1994 Economic Report of the President [3] provides a list of such situations in the post World War II United States). On the other hand, examples to the contrary are numerous, e.g., the luxury tax on boats introduced in the 1991 budget deal in the United States resulted in a dramatic fall of the revenues generated from taxes on such sales, as nearly the entire luxury boat industry in the United States disappeared. At this point we need to ask ourselves a rather fundamental question -- how do revenues change as tax rates increase? Is the relationship chaotic, or is there continuity in the change? We believe that no matter how dramatic the changes, the overall relationship is continuous, i.e., an infinitely small change in tax rates results in nearly infinitely small change in tax revenues. We should note that Gardner [4] challenges that premise on the basis of data from various periods, pointing out chaotic nature of the relationships revealed by the data. However, we should observe that it is quite unreasonable to expect the relationship between taxes collected and tax rates to be always identical. Over time, the relationship must, and will, change. On the other hand, assuming that there is no current relationship whatsoever, as Gardner proclaims, is quite unreasonable and not justified by any rational premise. We believe that there is a relationship, that it can be expressed in a functional form, and that the functional form should also be analyzed from a longer-term perspective, i.e., tax revenues should be studied in relation to tax rates *after* all of the information about changes in tax statutes has been absorbed by the economy.

Therefore, we begin with the function illustrating the relationship of tax rate  $T$  to the total tax revenue collected  $R(T)$  expressed in a graph such as the one presented in Figure 1 below:



**Figure 1**

### **The Laffer Curve Revisited**

We have a function  $R(T)$ , where  $T$  is the tax rate (between 0 and 1, i.e., between 0% and 100%) and  $R$  is the resulting total tax revenue. This function equals zero for  $T = 0\%$  and for  $T = 100\%$ . Furthermore, this function assumes positive values for values of  $T$  between 0% and 100%. By the Rolle's Theorem, the function achieves a local (and global) maximum for some number between 0% and 100%. This is one of the main points of the idea of the Laffer Curve. When setting tax rates, government should not waste its citizens efforts by allowing the tax rate to exceed the one which generates the highest revenues. The revenue-maximizing tax rate, let us call it  $T_{\max}$ , is, when compared to any higher tax rate (but not when compared to lower tax rates), Pareto efficient: both the government and the citizens are better off with tax rate set at  $T_{\max}$  than with any  $T$  greater than  $T_{\max}$  (this assumes that we do not treat government as agents for the citizens, and such assumption may be debatable, but this controversy is not crucial to our analysis). Granted, the question of what the exact value of  $T_{\max}$  is, remains unresolved. We believe there is no simple answer to this

question, and it can be examined empirically. It is also quite likely that the actual value of  $T_{\max}$  changes over time and between various cultures. We will leave such examination for another study. However, one can gain some insight by examining the theoretical function itself.

Let us note that it seems quite natural to us that different countries may have different values of  $T_{\max}$ , depending on their historical and cultural background, as well as the value they place on government services, and that the value of  $T_{\max}$  may evolve over time, depending on economic circumstances. The same can be said of the entire Laffer Curve. One should not, indeed, speak of a single Laffer Curve, but rather of a Laffer Process, describing the evolution of the economy's response to taxes. Furthermore, as the powerful work of the Rational Expectations school indicates, we must allow for the rational adaptation of human behavior to the tax incentives, thus a short run Laffer Curve may differ significantly from a more efficient long run Laffer Curve. These issues do require further study which may become quite involved. There is, however, an insight gained by allowing dynamic structure to the Laffer Curve, which to our surprise has remained largely ignored. Allow us to present it here.

Browning and Browning (1987, pp. 456-458) point out that the tax rate corresponding to the maximum of revenue cannot be optimal, as the intersection of the marginal revenue function and marginal benefit of government services function must occur at a lower tax rate. The argument is quite basic, and it relies on the standard cost/benefit analysis. However, Browning and Browning clearly state that their conclusions depend on knowledge of the benefit of government services function, and cannot be derived from the Laffer Curve alone. We propose that the Laffer Curve can, and indeed does, provide information about optimal level of taxation. Furthermore, the Laffer Curve can be, to a degree, empirically studied, while the benefit function is purely theoretical in nature and cannot be studied empirically. Indeed, we submit that the Laffer Curve may be the only direct manifestation of the benefit function available to us. This, we believe, may be one of the key starting points of our analysis.

The model we are applying here contains some simplifying assumptions. We do ignore any underground economy for a moment, and concern ourselves with that portion of economic activities which is, indeed, visible to the tax authorities and "available for taxation." Furthermore, we will assume that the tax collected is a simple proportion  $T$  of income produced  $I$ , so that

$$R(T) = T \cdot I(T). \quad (1)$$

In a neighborhood of the point where  $R(T)$  reaches its maximum, since this function has the rate of change of zero, it is nearly constant, so that the function  $I(T)$  must be approximately proportional (with a positive coefficient) to  $\frac{1}{T}$ . This in turn implies that the derivative of the function  $I(T)$  at the point  $T_{\max}$  is approximately proportional to  $-\frac{1}{T^2}$ . Therefore, in a neighborhood of the point of maximum revenue, the total national income is a decreasing function of the tax rate. If taxes are set near the value  $T_{\max}$ , revenues are maximized, but as taxes rise, national income falls. One must be then naturally inclined to ask: given the nature of this relationship, what tax rates would result in maximization of national income?

Given the relationship (1), we have:

$$R(T) = I(T) + T \cdot I'(T). \quad (2)$$

If the maximum of  $I(T)$  exists, then  $I'(T) = 0$  there, resulting in:

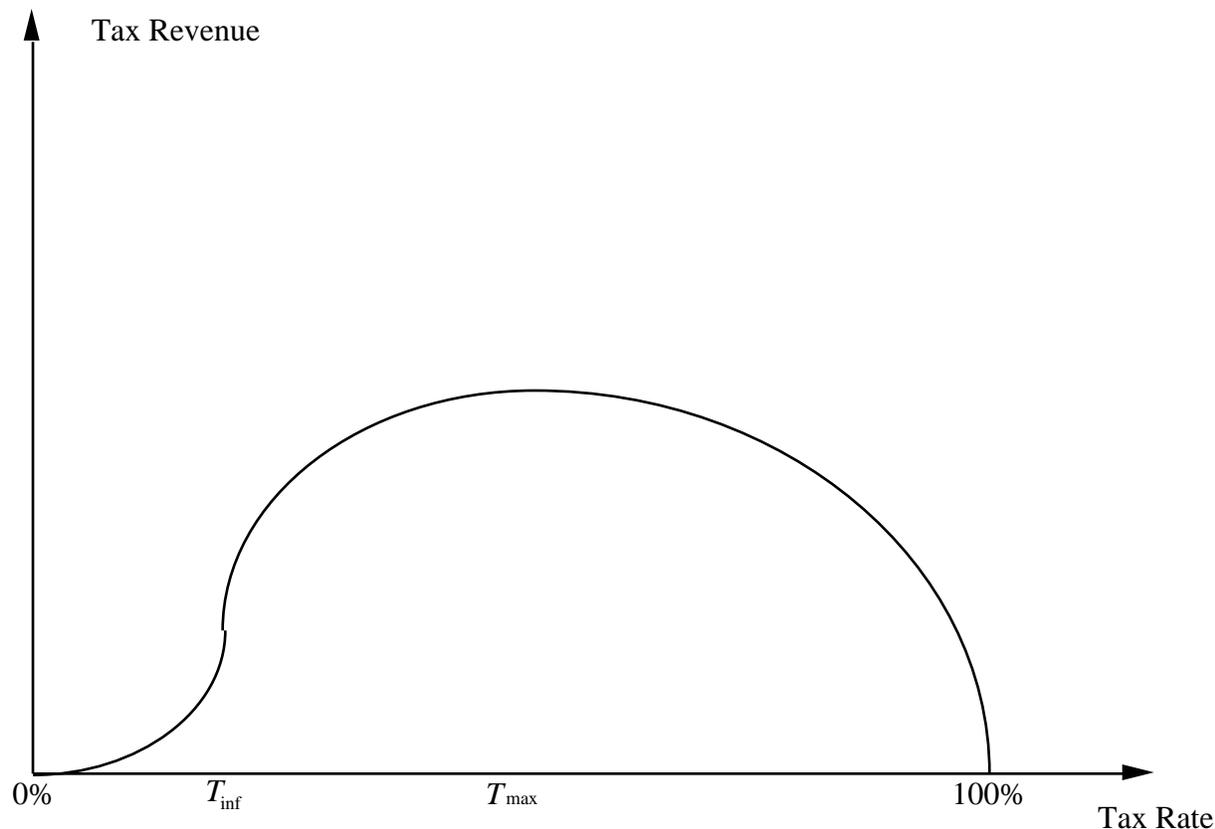
$$R(T) = I(T) = \frac{R(T)}{T}. \quad (3)$$

This simply says that national income is maximized where tax rate is such that the marginal revenue from new taxes equals the average revenue from old taxes, in surprisingly microeconomic terms. In graphical terms, this means the tangent to the curve at a point  $T$  has the same slope as the secant line. For a curve drawn in Figure 1, this happens only at  $T = 0$ .

What is wrong with this picture? Figure 1, which shows the shape of the Laffer Curve most often displayed in macroeconomic textbooks, claims that government always has diminishing returns to scale. Any increase in tax rates results in a proportionally smaller increase in the total tax revenues, and beyond  $T_{\max}$ , causes losses of tax revenue. We believe such a model to be extremely unrealistic. The institution of government exists for a reason. Low tax rates usually mean "emerging governments", i.e., governments establishing the rule of law and strengthening the social order. Such activities encourage enterprise and generally result in better enforcement of contracts and all agreements in the society. Thus it is perfectly reasonable to assume that at low tax rates, increases in tax rates should result in ever more increasing economic activity, and disproportionately high increases in tax revenue collected. How low is low? Again, this is most likely a culture-specific, time-dependent, and dynamic question. However, one can hardly argue that any tax rate over 50% is low -- in fact we are quite convinced that beyond 50% marginal tax rate the Laffer Curve must be falling: why work for an extra dollar if you can make more by avoiding taxes on the last dollar earned? On the other hand, most traditions indicate that marginal tax rates below 10% are considered natural or even harmless -- after all, 10% tithe is quite universally expected. We would certainly argue that tax rates below 10% can result in government enjoying increasing returns to

scale. This means that the Laffer Curve is not always concave down. It starts out being concave up. As tax burden rises more and more, this phenomenon subsides, and eventually gives in to negative convexity (that is, the second derivative of the function  $R(T)$  starts out as positive, and only under the heavy burden of rising tax rates does it become negative). Once this negative convexity is achieved, any increases in tax rates will only strengthen it, leading eventually to the maximum of the function  $R(T)$ , and to its eventual demise. Let us notice something here. The point at which  $R(T)$  changes the sign of its second derivative is called *the inflection point* of the curve. It represents the point at which government no longer has increasing returns to scale. Let us call it  $T_{inf}$ .

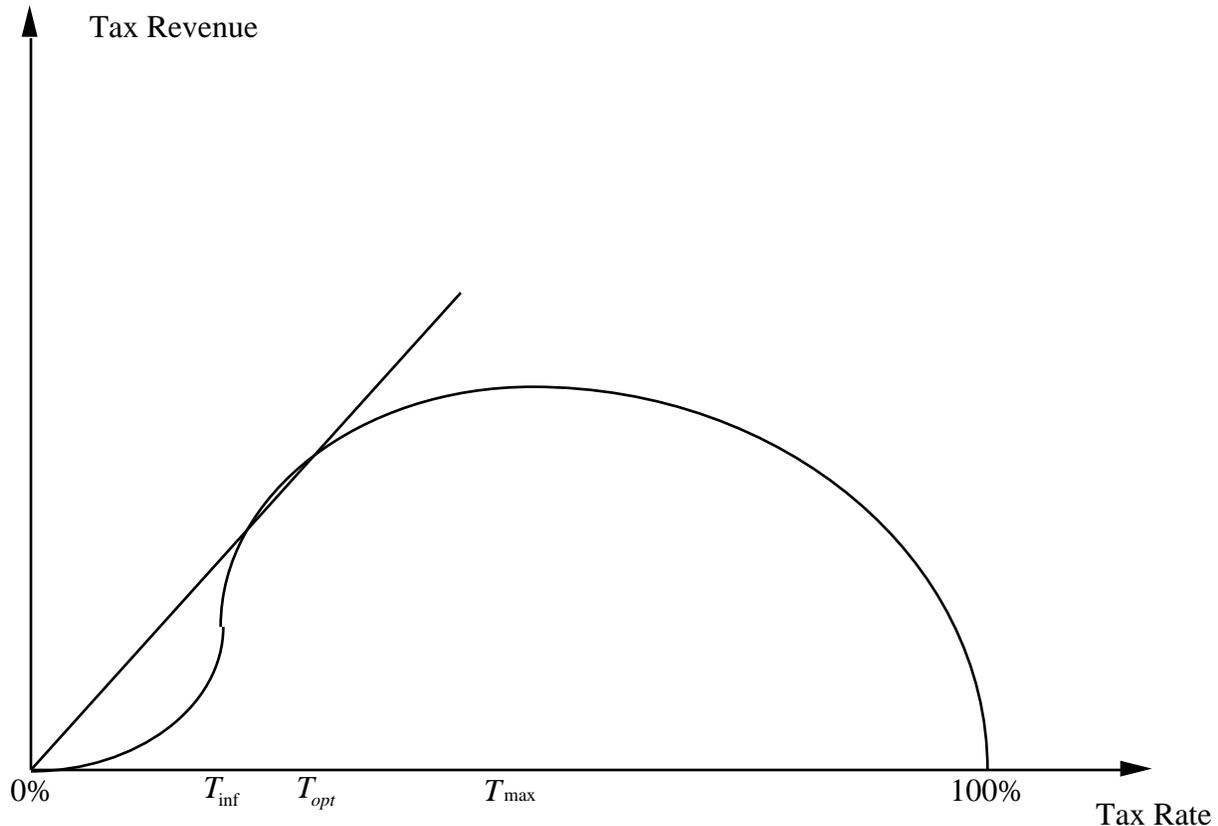
Our conclusion means that the graph of the Laffer Curve is dramatically different than the one presented in Figure 1. We show it Figure 2 below.



**Figure 2**

If the premise of existence of the inflection point of the Laffer Curve is accepted, it does arrive with consequences, as shown in the previous mathematical derivation. It implies that there is a tax rate, denoted here by  $T_{opt}$ , at which, in relation to all other possible tax rates, even though the government's total revenues are not maximized, the output subject to taxation is. This means that

setting tax rates above the point  $T_{opt}$  results in increases in government revenues which are more than offset by losses to the national economy. The point  $T_{opt}$  is determined as the one where the tangent to the Laffer Curve coincides with the secant line from the origin to the point on the graph. This is illustrated in Figure 3 below.



**Figure 3**

This is a relationship that labor economists are quite familiar with. When considering increases in labor force, we add labor until marginal productivity of new labor equals average productivity of existing labor. When applied to government, it states that we increase payments only for so much of government that for any new 1% in tax rates we would get as much government as we get on average from any existing 1% in tax rates.

Our analysis implies that setting the tax rates is a public policy decision which can be viewed from three different perspectives:

- that of the society and its perceived need for the government;
- that of the national economy and its growth;
- that of the government and its need for revenue.

The three points identified in Figure 3 are optimal relative to the three criteria identified above, respectively.

### Burden of Taxes

Note that the secant line in Figure 3 illustrates a situation of taxation as ideally perceived by accounting -- revenues are simply directly proportional to income taxed. The Laffer Curve, on the other hand, represents the reality caused by the taxpayers' response to taxes put on them. From the point of view of taxpayer a good measure of tax burden is not the tax rate, i.e., the ratio of taxes to gross income, but rather the ratio of taxes amount to the after-tax income. This marginal tax burden can be measured by a function  $B$  which relates to the tax rate  $T$  through the formula:

$$\frac{dB}{dT} = \frac{1}{1-T} \quad (4)$$

representing the fact that any increase of tax rate of 1% (absolute) is a larger tax burden in relation to the remaining after-tax income of the taxpayer. (4) is an elementary differential equation which gives the tax burden function as

$$B(T) = -\ln(1-T), \quad (5)$$

with  $B(0) = 0$ , and  $\lim_{T \rightarrow 1} B(T) = +\infty$ , as expected. Note that the Taylor series expansion of (5) allows for approximation  $B(T) = T$  for small values of  $T$ , but *only* for small values of it.

Our analysis of the Laffer Curve has effectively suggested criteria other than maximization of tax revenue that might be used in setting the tax rates. Possible criteria are:

- *Effectiveness of Government*: Increase the tax rate as long as marginal tax revenues exceed average tax revenues;
- *Utility of Government*: Increase the tax rate as long as such increases bring increasing returns to scale.

The first criterion gave the optimality condition (3) and the tax rate  $T_{opt}$ . The second criterion implies seeking the inflection point:

$$R''(T) = 0. \quad (6)$$

(6) gives the lower  $T_{inf}$  optimal tax rate. But these criteria can also be analyzed in terms of the tax burden (4). If we use the effectiveness criterion analogous to (3), i.e., we seek average tax revenue equal to marginal tax revenue, we arrive at:

$$\frac{dR}{dB} = \frac{dR}{dT} \frac{dT}{dB} = R'(T)(1-T) \quad (7)$$

and

$$\frac{R}{B} = \frac{R(T)}{-\ln(1-T)} \quad (8)$$

so that

$$\frac{R(T)}{-\ln(1-T)} = R(T)(1-T), \quad (9)$$

i.e.,

$$R(T) = \frac{R(T)}{-\ln(1-T)(1-T)} > \frac{R(T)}{T}. \quad (10)$$

The inequality in (10) is implied by the fact that:

$$\begin{aligned} \frac{T}{1-T} &= T + T^2 + T^3 + \dots > T + \frac{1}{2}T^2 + \frac{1}{3}T^3 + \dots = \\ &= \int_0^T \frac{1}{1-t} dt = -\ln(1-T) \end{aligned} \quad (11)$$

(10) implies that at the point of optimality with respect to the tax burden, marginal tax revenues exceed average tax revenues, i.e., tax rate which optimizes the effectiveness of government as perceived by the tax burden of the taxpayer is lower than when the perception of the overall economy (private and public) is combined.

If the criterion of the utility of government is applied, we arrive at:

$$R(T(B)) = R(T)(T(B))^2 + R(T)T(B) = 0 \quad (12)$$

resulting in:

$$R(T) = \frac{R(T)}{1-T}. \quad (13)$$

(13) means that the optimal value of  $T$  occurs for a positive value of  $R(T)$ , i.e., while the government has still positive returns to scale. Again, optimal tax policy occurs at a lower tax rate for the private sector than for the overall economy (private and public) combined.

Let us note one more optimization process that can be performed here. If we simply wanted to maximize national after-tax income, we would seek to maximize  $I(T) - R(T)$ . The maximum occurs where  $I(T) = R(T)$ , i.e., where marginal private benefit from taxes equals marginal tax revenue to the government. Given the existence of an inflection point of the Laffer Curve (of which we are quite convinced), this criterion implies that tax rates should be increased as long as their private benefit exceeds tax revenues earned. This criterion may be abstract in nature, but it is, in fact,

a straightforward pricing mechanism for government services. This optimization criterion gives the following optimality condition:

$$R(T) = \frac{R(T)}{T} \frac{1}{1-T}, \quad (14)$$

and again produces the value of  $T$  to the left of  $T_{opt}$ . Thus all three alternative criteria, applied in terms of after-tax income instead of the overall income, call for a smaller than  $T_{opt}$  value of the effective tax rate.

### Conclusions

We believe that the observations of this note, although theoretical in nature, can become a helpful tool in public policy decisions. We have initially assumed that no “underground economy” exists, and that taxes are collected through a simple proportional flat tax on income. These assumptions are not essential for our conclusions. All tax collections effectively cut into taxpayers’ incomes, and decreasing returns to scale from very high taxes result precisely from changes in behavior of taxpayers in response to the tax system, including moving “underground”. The dividing line between “underground” and regular tax avoidance, although legally sharp, in practice is less clear. Income is produced, and then it is taxed -- what portion of it should be collected? How do we decide what the best answer to this question is? We believe that our note provides some guidelines for possible practical answer to such questions.

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